

Algebra

CHAPTER 14

Probability II

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CONDITIONAL PROBABILITY

Conditional probability is a measure of the probability of an event (some particular situation occurring) given that (by assumption, presumption, assertion or evidence) another event has occurred. If the event of interest is A and the event B is known or assumed to have occurred, “the conditional probability of A given B ”, or “the probability of A under the condition B ”, is usually written as

$$P(A|B), P(A/B) \text{ or } P\left(\frac{A}{B}\right).$$

Consider the following examples.

- (i) Let a bag contain 2 red balls and 3 black balls. One ball is drawn from the bag and this ball is not replaced in the bag. Then a second ball is drawn from the bag.

Let B denote the event of occurrence of a red ball in the first draw and A denote the event of occurrence of a black ball in the second draw.

When a red ball has been drawn in the first draw, the number of balls left is 4 and out of these four balls, 1 is a red ball and 3 are black balls.

$\therefore P(A/B)$ = Probability of occurrence of a black ball in the second draw when a red ball has been drawn in the first

$$\text{draw} = \frac{3}{4}$$

- (ii) When a dice is thrown, sample space $S = \{1, 2, 3, 4, 5, 6\}$
Let A = Event of occurrence of a number greater than 4
= $\{5, 6\}$

B = Event of occurrence of an odd number = $\{1, 3, 5\}$

Then

$P(A/B)$ = Probability of occurrence of a number greater than 4 when an odd number has already occurred ... (1)

Here, it is known that an odd number has occurred which means that one of 1, 3 and 5 has occurred. Out of these three numbers, only 5 is greater than 4.

So, when an odd number has occurred, total number of cases is only 3 (not 6) and favourable number of cases is 1.

$$\text{Hence, from (1), } P(A/B) = \frac{1}{3} = \frac{n(A \cap B)}{n(B)}$$

Clearly, for event A/B , B is sample space and $A \cap B$ is set of favourable cases.

In the example given above,

$P(B/A)$ = Probability of occurrence of an odd number when a number greater than 4 has already occurred

$$= \frac{n(B \cap A)}{n(A)} = \frac{1}{2}$$

Here, $A = \{5, 6\}, B \cap A = \{5\}$

$$\therefore n(A) = 2, \text{ and } n(B \cap A) = 1.$$

It is to be noted that $P(A/B)$ may or may not be equal to $P(B/A)$.

Now, let us formally define conditional probability.

If A and B are any two events, $B \neq \phi$, then $P(A/B)$ denotes the conditional probability of occurrence of event A when B has already occurred

$$P(A/B) = \frac{n(A \cap B)}{n(B)}$$

PROPERTIES OF CONDITIONAL PROBABILITY

If A and B are events of a sample space S of an experiment, then

- (i) $P(S/A) = P(A/A) = 1$

Proof:

$$P(S/A) = \frac{n(S \cap A)}{n(A)} = \frac{n(A)}{n(A)} = 1$$

$$P(A/A) = \frac{n(A \cap A)}{n(A)} = \frac{n(A)}{n(A)} = 1$$

- (ii) If $P(C) \neq 0$, then

$$P((A \cup B)/C) = P(A/C) + P(B/C) - P((A \cap B)/C)$$

Proof:

$$\begin{aligned} P((A \cup B)/C) &= \frac{n((A \cup B) \cap C)}{n(C)} \\ &= \frac{n((A \cap C) \cup (B \cap C))}{n(C)} \\ &= \frac{n(A \cap C) + n(B \cap C) - n((A \cap B) \cap C)}{n(C)} \\ &= \frac{n(A \cap C)}{n(C)} + \frac{n(B \cap C)}{n(C)} - \frac{n((A \cap B) \cap C)}{n(C)} \\ &= P(A/C) + P(B/C) - P((A \cap B)/C) \end{aligned}$$

- (iii) $P(A'/B) = 1 - P(A/B)$, if $P(B) \neq 0$.

Proof:

From Property 1, we know that

$$P(S/B) = 1$$

$$\Rightarrow P((A \cup A')/B) = 1 \quad (\text{As } A \cup A' = S)$$

$$\Rightarrow P(A/B) + P(A'/B) = 1$$

(As A and A' are disjoint events)

$$\Rightarrow P(A'/B) = 1 - P(A/B)$$

ILLUSTRATION 14.1

Two dice are thrown. What is the probability that the sum of the numbers appearing on the two dice is 11, if 5 appears on the first?

Sol. Let event B is “5 appears on the first dice”.

$$\therefore B = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$$

Also, let event A is “sum of the numbers appearing is 11”.

$$\therefore A = \{(5, 6), (6, 5)\}$$

$$\therefore A \cap B = \{(5, 6)\}$$

$$\text{Therefore, } P(A/B) = \frac{P(A \cap B)}{n(B)} = \frac{1}{6}$$

ILLUSTRATION 14.2

If $P(A) = 0.8$, $P(B) = 0.5$, and $P(B/A) = 0.4$, find

- (i) $P(A \cap B)$ (ii) $P(A/B)$ (iii) $P(A \cup B)$

Sol. It is given that $P(A) = 0.8$, $P(B) = 0.5$, and $P(B/A) = 0.4$

(i) $P(B/A) = 0.4$

$$\therefore \frac{P(A \cap B)}{P(A)} = 0.4$$

$$\text{or } \frac{P(A \cap B)}{0.8} = 0.4$$

$$\text{or } P(A \cap B) = 0.32$$

(ii) $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$\text{or } P(A/B) = \frac{0.32}{0.5} = 0.64$$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\text{or } P(A \cup B) = 0.8 + 0.5 - 0.32 = 0.98.$$

ILLUSTRATION 14.3

If two events A and B are such that $P(A) = 0.3$; $P(B) = 0.4$ and $P(A' \cap B') = 0.5$, then find the value of $P(B/(A \cup B'))$.

$$\begin{aligned} \text{Sol. } P(B/(A \cup B')) &= \frac{P(B \cap (A \cup B'))}{P(A \cup B')} \\ &= \frac{P[(B \cap A) \cup (B \cap B')]}{P(A) + P(B') - P(A \cap B')} \\ &= \frac{P(A \cap B)}{P(A) + P(B') - P(A \cap B')} \end{aligned}$$

$$\text{Now, } P(A' \cap B') = 0.5$$

$$\therefore 1 - P(A \cup B) = 0.5$$

$$\Rightarrow P(A \cup B) = 0.5$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = 0.5$$

$$\Rightarrow P(A \cap B) = 0.3 + 0.4 - 0.5 = 0.2$$

$$\Rightarrow P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.2 = 0.1$$

$$P(B/(A \cup B')) = \frac{0.2}{0.3 + 0.6 - 0.1} = \frac{0.2}{0.8} = \frac{1}{4}$$

ILLUSTRATION 14.4

A coin is tossed three times, where

(i) A : head on third toss, B : heads on first two tosses

(ii) A : at least two heads, B : at most two heads

(iii) A : at most two tails, B : at least one tail

In each case find $P(A/B)$.

Sol. If a coin is tossed three times, then the sample space S is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$\therefore n(S) = 8.$$

(i) $A = \{HHH, HTH, THH, TTH\}$

$$B = \{HHH, HHT\}$$

$$\therefore A \cap B = \{HHH\}$$

$$P(B) = \frac{2}{8} = \frac{1}{4} \text{ and } P(A \cap B) = \frac{1}{8}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{4}{8} = \frac{1}{2}$$

(ii) $A = \{HHH, HHT, HTH, THH\}$

$$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$\therefore A \cap B = \{HHT, HTH, THH\}$$

$$\text{Clearly, } P(A \cap B) = \frac{3}{8} \text{ and } P(B) = \frac{7}{8}$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}$$

(iii) $A = \{HHH, HHT, HTT, HTH, THH, THT, TTH\}$

$$B = \{HHT, HTT, HTH, THH, THT, TTH, TTT\}$$

$$\therefore A \cap B = \{HHT, HTT, HTH, THH, THT, TTH\}$$

$$P(B) = \frac{7}{8} \text{ and } P(A \cap B) = \frac{6}{8}$$

$$\text{Therefore, } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$$

ILLUSTRATION 14.5

A dice is thrown three times and the sum of the thrown numbers is 15. Find the probability for which number 4 appears in first throw.

Sol. We have to find the conditional probability of sum 15 when 4 appears first.

Let the event of getting sum of three thrown numbers as 15 be A and the event of appearing 4 be B .

So, we have to find $P(A/B)$.

$$\text{Now, } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Event $(A \cap B)$ is “4 appears in first throw and sum of numbers is 15”.

$$\therefore A \cap B = \{(4, 5, 6), (4, 6, 5)\}$$

$$\therefore P(A/B) = \frac{2}{36} = \frac{1}{18}$$

ILLUSTRATION 14.6

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

- (i) the youngest is a girl, (ii) at least one is a girl?

Sol. Let b and g represent the boy and the girl child, respectively. If a family has two children, the sample space will be

$$S = \{(b, b), (b, g), (g, b), (g, g)\}$$

Let A be the event that both children are girls. Therefore,

$$A = \{(g, g)\}$$

- (i) Let B be the event that the youngest child is a girl. Therefore,

$$B = \{(b, g), (g, g)\}$$

$$\Rightarrow A \cap B = \{(g, g)\}$$

$$\therefore P(B) = \frac{2}{4} = \frac{1}{2} \text{ and } P(A \cap B) = \frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by $P(A/B)$.

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2}$$

- (ii) Let C be the event that at least one child is a girl. Therefore,

$$C = \{(b, g), (g, b), (g, g)\}$$

$$\Rightarrow A \cap C = \{(g, g)\}$$

$$\Rightarrow P(C) = \frac{3}{4} \text{ and } P(A \cap C) = \frac{1}{4}$$

The conditional probability that both are girls, given that at least one child is a girl, is given by $P(A|C)$. Therefore,

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{1/4}{3/4} = \frac{1}{3}$$

ILLUSTRATION 14.7

Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a dice. Find the conditional probability of the event 'the dice shows a number greater than 4' given that there is at least one tail.

Sol. According to the question,

$$S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

In the above set, there are 8 elementary events, but all are not equally likely.

However, events (H, H) and (H, T) are equally likely. Each event has probability $\frac{1}{4}$.

Events $(T, 1), (T, 2), \dots, (T, 6)$ are equally likely and each event has probability $\frac{1}{12}$.

Let event A be "the dice shows a number greater than 4"

And event B be "there is at least one tail".

$$\therefore A = \{(T, 5), (T, 6)\}$$

$$\text{and } B = \{(H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

$$\therefore A \cap B = \{(T, 5), (T, 6)\}$$

$$\therefore \text{Required probability, } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{12} + \frac{1}{12}}{\frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}} = \frac{1/6}{3/4} = \frac{2}{9}$$

ILLUSTRATION 14.8

A box contains 10 mangoes out of which 4 are rotten. Two mangoes are taken out together. If one of them is found to be good, then find the probability that the other is also good.

Sol. Let A be the event that the first mango is good, and B be the event that the second one is good. Then, required probability is

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Now, probability that both mangoes are good is

$$P(A \cap B) = \frac{{}^6C_2}{{}^{10}C_2}$$

Probability that first mango is good is

$$P(A) = \frac{{}^6C_2}{{}^{10}C_2} + \frac{{}^6C_1 \times {}^4C_1}{{}^{10}C_2}$$

$$\text{Hence, } P(B/A) = \frac{{}^6C_2}{{}^6C_2 + {}^6C_1 \times {}^4C_1} = \frac{15}{15 + 24} = \frac{5}{13}$$

CONCEPT APPLICATION EXERCISE 14.1

- Three coins are tossed. If one of them shows tail, then find the probability that all three coins show tail.
- If two events A and B are such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then find the value of $P[B/(A \cup B')]$.
- In a single throw of two dice what is the probability of obtaining a number greater 7, if 4 appears on the first dice?
- A coin is tossed three times in succession. If E is the event that there are at least two heads and F is the event in which first throw is a head, then find $P(E/F)$.
- Consider a sample space " S " representing the adults in a small town who have completed the requirements for a collage degree. They have been categorized according to sex and employment as follows:

	Employed	Unemployed
Male	460	40
Female	140	260

An employed person is selected at random. Find the probability that the chosen one is a male.

ANSWERS

1. $1/7$ 2. $1/4$ 3. $1/2$ 4. $3/4$ 5. $23/30$

MULTIPLICATION THEOREM OF PROBABILITY

Let A and B be two events associated with a sample space S . Clearly, the set $A \cap B$ denotes the event that both A and B have occurred. In other words, $A \cap B$ denotes the simultaneous occurrence of the events A and B . The event $A \cap B$ is also written as AB . Very often we need to find the probability of the event AB . For example, in the experiment of drawing two cards one after the other, we may be interested in finding the probability of the event 'a spade and a diamond'. The probability of event AB is obtained by using the conditional probability.

We know that the conditional probability of event A given that B has occurred is denoted by $P(A/B)$ and is determined by

$$P(A/B) = \frac{n(A \cap B)}{n(B)}, P(B) \neq 0$$

$$\therefore P(A \cap B) = P(A/B)P(B) \quad (1)$$

$$\text{Also, } P(B/A) = \frac{n(B \cap A)}{n(A)}, P(A) \neq 0$$

$$\therefore P(A \cap B) = P(B/A)P(A) \quad (2)$$

From (1) and (2), we also get

$$P(A \cap B) = P(A/B)P(B) = P(B/A)P(A), \text{ if } P(A), P(B) \neq 0.$$

This result is known as multiplication rule of probability.

MULTIPLICATION RULE OF PROBABILITY OF MORE THAN TWO EVENTS

If A , B and C are three events of sample space, we have

$$\begin{aligned} P(A \cap B \cap C) &= P(A)P(B/A)P(C/(A \cap B)) \\ &= P(A)P(B/A)P(C/AB). \end{aligned}$$

Similarly, the multiplication rule of probability can be extended for four or more events.

ILLUSTRATION 14.9

An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that first ball is black and second ball is white?

Sol. Let A and B denote, respectively, the events that first ball drawn is black and second ball drawn is white.

We have to find $P(A \cap B)$ or $P(AB)$.

$$P(A \cap B) = P(A)P(B/A)$$

$$\text{Now, } P(A) = P(\text{black ball in first draw}) = \frac{10}{15} = \frac{2}{3}$$

Also, given that the first ball drawn is black, i.e., event A has occurred. Now, there are 9 black balls and 5 white balls left in the urn. Therefore, the probability that the second ball drawn is white, given that the ball in the first draw is black, is nothing but the conditional probability of B given that A has occurred.

$$\therefore P(B/A) = \frac{5}{14}$$

Therefore,

$$P(A \cap B) = P(A)P(B/A) = \frac{2}{3} \times \frac{5}{14} = \frac{5}{21}.$$

ILLUSTRATION 14.10

Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first, second and third cards are jack, queen and king, respectively?

Sol. Let events J be first drawn card is jack, Q be second drawn card is queen and K be third drawn card is king.

We have to find the value of $P(J \cap Q \cap K)$

$$\text{Now, } P(J \cap Q \cap K) = P(J) \times P(Q/J) \times P(K/(J \cap Q))$$

$$P(J) = \frac{4}{52} = \frac{1}{13}$$

$$\begin{aligned} P(Q/J) &= P(\text{drawing queen card if jack card has been drawn}) \\ &= \frac{4}{51} \end{aligned}$$

$$\begin{aligned} P(K/(J \cap Q)) &= P(\text{drawing king card if jack and queen cards have been drawn}) \\ &= \frac{4}{50} = \frac{2}{25} \end{aligned}$$

$$\begin{aligned} \text{So, } P(J \cap Q \cap K) &= P(J) \times P(Q/J) \times P(K/(J \cap Q)) \\ &= \frac{1}{13} \times \frac{4}{51} \times \frac{2}{25} \end{aligned}$$

ILLUSTRATION 14.11

One of the ten available keys opens the door. If we try the keys one after another, then find the following

- the probability that the door is opened in the first attempt.
- the probability that the door is opened in the second attempt.
- the probability that the door is opened in the third attempt.
- the probability that the door is opened in the tenth attempt.

Sol. Let A_i be the event that door is opened in i th attempt.

- Since out of 10 keys only one key is correct, the probability that the door is opened in the first attempt is same as probability that the door is opened with randomly chosen key.

$$\text{So, } P(A_1) = \frac{1}{10}.$$

- If door is opened in the second attempt, then the first attempt must have failed.

$$\begin{aligned} \text{So, } P(\text{door is opened in second attempt}) \\ &= P(A_1' \cap A_2) = P(A_1')P(A_2/A_1') \end{aligned}$$

$$P(A_1') = P(\text{first attempt is fail}) = \frac{9}{10}$$

$$\begin{aligned} P(A_2/A_1') &= P(\text{second attempt is success when it is known that first attempt is fail}) \\ &= P(\text{door is opened with randomly chosen key from 9 keys}) \end{aligned}$$

$$= \frac{1}{9}$$

$$\text{So, } P(A_1' \cap A_2) = P(A_1')P(A_2/A_1') = \frac{9}{10} \times \frac{1}{9} = \frac{1}{10}$$

(iii) P (door is opened in third attempt)

$$\begin{aligned} &= P(A_1' \cap A_2' \cap A_3) \\ &= P(A_1') P(A_2'/A_1') P(A_3/(A_2' \cap A_1')) \\ &= \frac{9}{10} \times \frac{8}{9} \times \frac{1}{8} = \frac{1}{10} \end{aligned}$$

(iv) P (door is opened in tenth attempt)

$$\begin{aligned} &= \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} \times \frac{6}{7} \times \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1 \\ &= \frac{1}{10} \end{aligned}$$

ILLUSTRATION 14.12

A bag contains ' W ' white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. Then find the probability that this procedure of drawing the balls will come to an end at the r th draw.

Sol. Procedure of drawing the balls has to end at the r th draw.

So, exactly two black balls are drawn in first $(r-1)$ draws and third black ball is drawn in r th draw.

Let event A be "In first $(r-1)$ draws, two black balls are drawn" and event B be " r th draw is black ball".

We have to find $P(A \cap B)$.

Now, $P(A \cap B) = P(A) P(B/A)$

$P(A) = P(2 \text{ black and } (r-3) \text{ white balls are drawn from } (W+3) \text{ balls})$

$$\begin{aligned} &= \frac{{}^3C_2 \cdot {}^WC_{r-3}}{{}^{W+3}C_{r-1}} \\ &= \frac{3!}{2!1!} \cdot \frac{W!}{(r-3)!(W-r+3)!} \\ &= \frac{(W+3)!}{(r-1)!(W-r+4)!} \\ &= \frac{3 \cdot (r-1)(r-2)(W-r+4)}{(W+3)(W+2)(W+1)} \end{aligned}$$

At the end of $(r-1)$ th draw, we would be left with 1 black and $(W-r+3)$ white balls.

$\therefore P(B/A) = P(\text{drawing the black ball at the } r\text{th draw})$

$$= \frac{1}{(W-r+4)}$$

Therefore,

$$\begin{aligned} P(A \cap B) &= P(A) P(B/A) \\ &= \frac{3(r-1)(r-2)(W-r+4)}{(W+3)(W+2)(W+1)(W-r+4)} \\ &= \frac{3(r-1)(r-2)}{(W+3)(W+2)(W+1)} \end{aligned}$$

CONCEPT APPLICATION EXERCISE 14.2

1. A bag contains 5 white and 3 black balls. 4 balls are successively drawn out and not replaced. What is the probability that they are alternately of different colours?
2. Cards are drawn one by one at random from a well shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. Then prove that probability that exactly n cards are drawn, is $\frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$, where $2 < n < 50$.
3. In a multiple choice question, there are four alternative answers of which one or more than one is correct. A candidate will get marks on the question only if he ticks the correct answer. The candidate decides to tick answers at random. If he is allowed up to three chances to answer the question, then find the probability that he will get marks on it.

ANSWERS

1. $\frac{1}{7}$
3. $\frac{1}{5}$

INDEPENDENT EVENTS

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of other events.

Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely. If A and B denote the events 'the card drawn is a heart' and 'the card drawn is a king' respectively, then

$$P(A) = \frac{13}{52} = \frac{1}{4} \text{ and } P(B) = \frac{4}{52} = \frac{1}{13}$$

Also, A and B ($A \cap B$) is the event 'the card drawn is the king of heart'.

$$\text{So, } P(A \cap B) = \frac{1}{52}$$

$$\text{Hence, } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{1/13} = \frac{1}{4}$$

Since $P(A) = \frac{1}{4} = P(A/B)$, we can say that the occurrence of event B has not affected the probability of occurrence of the event A .

We also have

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/52}{1/4} = \frac{1}{13} = P(B)$$

So, occurrence of event A has not affected the probability of occurrence of event B . Thus, A and B are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

Thus, two events A and B are independent if $P(A/B) = P(A)$ and $P(B/A) = P(B)$.

Also, $P(A \cap B) = P(A/B)P(B) = P(A)P(B)$.

This is the condition for two events A and B to be independent.

Therefore, to prove that two events A and B are independent, it is sufficient to prove that $P(A \cap B) = P(A)P(B)$. If $P(A \cap B) \neq P(A)P(B)$, then events are said to be dependent.

Three events A , B and C are said to be mutually independent, if $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, $P(B \cap C) = P(B)P(C)$ and $P(A \cap B \cap C) = P(A)P(B)P(C)$. If at least one of these is not true for three given events, we say that the events are not independent.

PROPERTIES OF INDEPENDENT EVENTS

If events A and B are independent then

$$(i) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = P(A) + P(B) - P(A) \cdot P(B)$$

(i) events A' (complement of A) and B are independent.

$$\begin{aligned} \text{As } P(A' \cap B) &= P(B) - P(A \cap B) \\ &= P(B) - P(A) \cdot P(B) \\ &= P(B)(1 - P(A)) \\ &= P(B) \cdot P(A') \end{aligned}$$

(ii) A and B' are independent.

(iii) A' and B' are independent.

$$\begin{aligned} \text{As } P(A' \cap B') &= P((A \cup B)') \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A) \cdot P(B) \\ &= (1 - P(A))(1 - P(B)) \\ &= P(A') \cdot P(B') \end{aligned}$$

ILLUSTRATION 14.13

A fair coin is tossed repeatedly. If tail appears on first four tosses, then find the probability of head appearing on fifth toss.

Sol. Since the trials are independent, so the probability that head appears on the fifth toss does not depend upon previous results of the tosses. Hence, required probability is equal to probability of getting head, i.e., $1/2$.

ILLUSTRATION 14.14

If $P(A/B) = P(A/B')$, then prove that A and B are independent.

Sol. $P(A/B) = P(A/B')$

$$\text{or } \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B')}{1 - P(B)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$\text{or } P(A \cap B)[1 - P(B)] = P(B) \cdot P(A) - P(B) \cdot P(A \cap B)$$

$$\text{or } P(A \cap B) = P(A) \cdot P(B)$$

Hence, A and B are independent.

ILLUSTRATION 14.15

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, “the number is even,” and B be the event, “the number is red”. Are A and B independent?

Sol. When a die is thrown, the sample space (S) is

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let A : the number is even = $\{2, 4, 6\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

B : the number is red = $\{1, 2, 3\}$

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

$$A \cap B = \{2\}$$

$$P(A \cap B) = \frac{1}{6}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6}$$

$$\Rightarrow P(A) \cdot P(B) \neq P(AB)$$

Therefore, A and B are not independent.

ILLUSTRATION 14.16

Three persons work independently on a problem. If the probabilities that they will solve it are $1/3$, $1/4$ and $1/5$, then find the probability that none can solve it.

Sol. Let three persons be A , B and C .

Clearly, the probabilities of solving the problem by A , B and C are independent.

Given that $P(A) = 1/3$, $P(B) = 1/4$ and $P(C) = 1/5$.

$\therefore P(\text{none can solve the problem})$

$$\begin{aligned} &= P(A' \cap B' \cap C') \\ &= P(A')P(B')P(C') \\ &= (1 - P(A))(1 - P(B))(1 - P(C)) \\ &= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \\ &= \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5} \end{aligned}$$

ILLUSTRATION 14.17

The probability of hitting a target by three marksmen A , B and C are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. Find the probability that one and only one of them will hit the target when they fire simultaneously

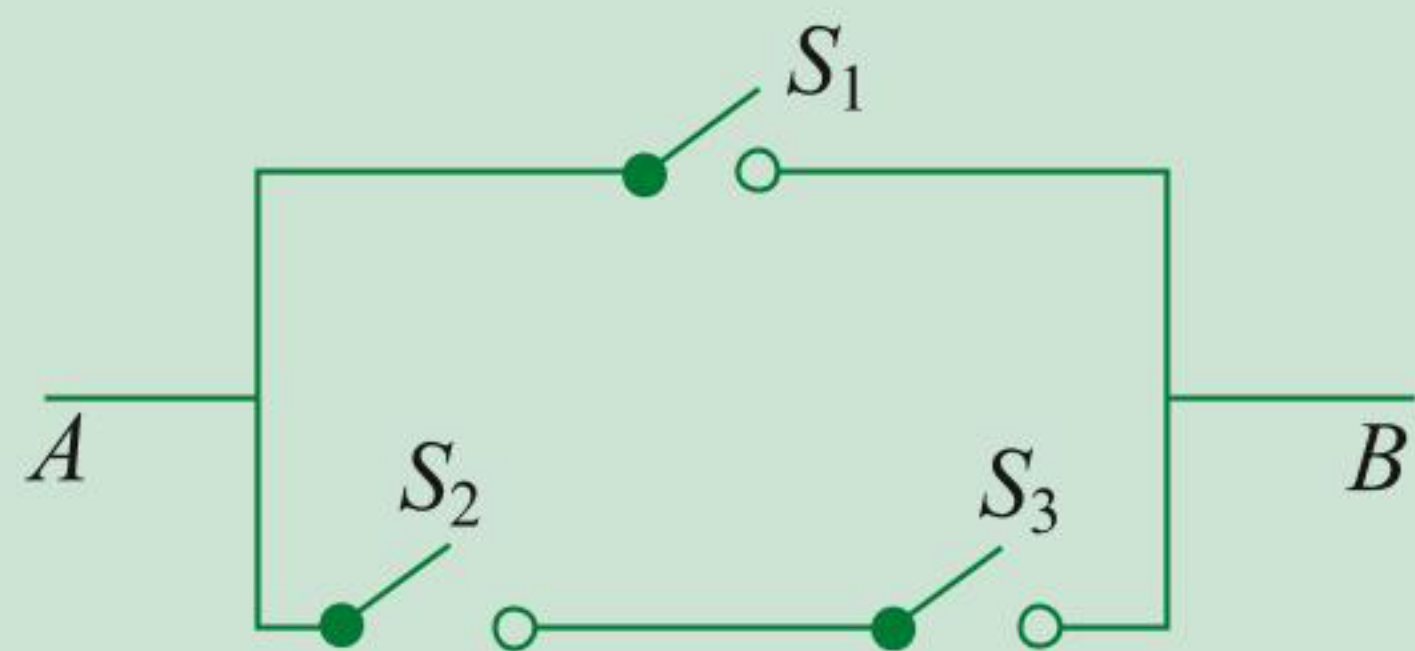
Sol. We have $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{4}$

Clearly, the probabilities of A , B , C hitting the target are independent. $P(\text{exactly one of them hitting the target})$

$$\begin{aligned} &= P(A \text{ is hitting and } B, C \text{ missing the target}) \\ &\quad + P(B \text{ is hitting and } A, C \text{ missing the target}) \\ &\quad + P(C \text{ is hitting and } A, B \text{ missing the target}) \\ &= P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C) \\ &= P(A)P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C) \\ &= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} = \frac{11}{24} \end{aligned}$$

ILLUSTRATION 14.18

An electrical system has open-closed switches S_1 , S_2 and S_3 as shown.



The switches operate independently of one another and the current will flow from A to B either if S_1 is closed or if both S_2 and S_3 are closed.

If $P(S_1) = P(S_2) = P(S_3) = \frac{1}{2}$, then find the probability that the circuit will work.

Sol. $P(S_1) = P(S_2) = P(S_3) = \frac{1}{2}$

Let E be event that “the current will flow”.

$$\begin{aligned} P(E) &= P((S_2 \cap S_3) \cup S_1) \\ &= P(S_2 \cap S_3) + P(S_1) - P(S_1 \cap S_2 \cap S_3) \\ &= P(S_2)P(S_3) + P(S_1) - P(S_1)P(S_2)P(S_3) \\ &= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{8} \end{aligned}$$

ILLUSTRATION 14.19

The odds against a certain event are 5:2 and the odds in favor of other event, independent of the former, are 6:5. Then find the probability that at least one of the events will happen.

Sol. $P\{\text{First event does not happen}\} = \frac{5}{5+2} = \frac{5}{7}$

$$P\{\text{Second event does not happen}\} = \frac{5}{5+6} = \frac{5}{11}$$

$$\therefore P\{\text{Both the events fail to happen}\} = \frac{5}{7} \times \frac{5}{11} = \frac{25}{77}$$

Therefore, the probability that at least one of the events will happen is

$$1 - P(\text{none of two happens}) = 1 - \frac{25}{77} = \frac{52}{77}$$

ILLUSTRATION 14.20

If four whole numbers taken at random are multiplied together, then find the probability that the last digit in the product is 1, 3, 7 or 9.

Sol. There are 10 digits 0, 1, 2, ..., 9 and any of which can occur in any number at the last place, i.e., at the unit place. It is obvious that if the last digit in any of the four numbers is 0, 2, 4, 5, 6, 8, then the product of any of such four numbers will not give a number having its last digit as 1, 3, 7, 9. Hence, it is necessary that the last digit in each of the four numbers must be any of the four digits 1, 3, 7, 9.

Thus, for each of the four numbers, the number of ways of selection of the last digit is 10. Favorable number of ways of selection of the last digit is 4.

Therefore, the probability that the last digit be any of the four numbers 1, 3, 7, 9 is $4/10 = 2/5$.

Hence, the required probability that the last digit in each of the four numbers is 1, 3, 7, 9 so that the last digit in their product is 1, 3, 7, 9 is $(2/5)^4 = 16/625$.

ILLUSTRATION 14.21

If A and B are two independent events, the probability that both A and B occur is $1/8$ and the probability that neither of them occurs is $3/8$. Find the probability of the occurrence of A .

Sol. We have,

$$P(A \cap B) = \frac{1}{8} \text{ and } P(\bar{A} \cap \bar{B}) = \frac{3}{8}$$

$$\therefore P(A)P(B) = \frac{1}{8} \text{ and } P(\bar{A})P(\bar{B}) = \frac{3}{8}$$

[$\because A$ and B are independent]

$$\text{Now, } P(\bar{A} \cap \bar{B}) = \frac{3}{8}$$

$$\Rightarrow 1 - P(A \cup B) = \frac{3}{8}$$

$$\text{or } 1 - (P(A) + P(B) - P(A \cap B)) = \frac{3}{8}$$

$$\text{or } 1 - (P(A) + P(B)) + \frac{1}{8} = \frac{3}{8}$$

$$\text{or } P(A) + P(B) = \frac{3}{4}$$

The quadratic equation whose roots are $P(A)$ and $P(B)$ is

$$x^2 - x\{P(A) + P(B)\} + P(A)P(B) = 0$$

$$\text{or } x^2 - \frac{3}{4}x + \frac{1}{8} = 0$$

$$\text{or } 8x^2 - 6x + 1 = 0$$

$$\text{or } x = \frac{1}{2}, \frac{1}{4}$$

ILLUSTRATION 14.22

The unbiased dice is rolled until a number greater than 4 appears. What is the probability that an even number of tosses is needed?

Sol. According to the question, number greater than 4 may appear in 2nd, 4th, 6th, ... trial.

$$\text{Probability of getting number greater than 4 in any trial} = \frac{2}{6} = \frac{1}{3}$$

$\therefore$ Probability of not getting the number greater than 4 in any trial

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Let A_i be the event that number greater than 4 occurs in the trial.

Now, result of each trial is independent of earlier trials.

So, required probability

$$\begin{aligned} &= P(A_1' \cap A_2) + P(A_1' \cap A_2' \cap A_3' \cap A_4) \\ &\quad + P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5' \cap A_6) + \dots \end{aligned}$$

$$\begin{aligned}
&= P(A_1') P(A_2) + P(A_1') P(A_2') P(A_3') (A_4) + \dots \\
&= \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \dots \\
&= \frac{\frac{2}{3} \times \frac{1}{3}}{1 - \frac{2}{3} \times \frac{2}{3}} \quad (\text{Sum of infinite G.P.}) \\
&= \frac{2}{5}
\end{aligned}$$

ILLUSTRATION 14.23

A pair of unbiased dices are rolled together till a sum of either 5 or 7 is obtained. Then find the probability that 5 comes before 7.

Sol. Let A denote the event that a sum of 5 occurs, B be the event that a sum of 7 occurs and C be the event that neither a sum of 5 nor a sum of 7 occurs.

We have

$$P(A) = \frac{4}{36} = \frac{1}{9}, P(B) = \frac{6}{36} = \frac{1}{6} \text{ and } P(C) = \frac{26}{36} = \frac{13}{18}$$

$$\begin{aligned}
\text{Thus, } P(A \text{ occurs before } B) &= \frac{1}{9} + \left(\frac{13}{18}\right) \times \frac{1}{9} + \left(\frac{13}{18}\right)^2 \frac{1}{9} + \dots \\
&= \frac{1/9}{1 - 13/18} = \frac{2}{5}
\end{aligned}$$

ILLUSTRATION 14.24

An unbiased normal coin is tossed n times. Let
 E_1 : event that both **Head and Tail** are present in n tosses.
 E_2 : event that the coin shows up **Head** atmost once.
Find the value of n for which E_1 and E_2 are independent.

Sol. $P(E_1) = 1 - [P(\text{all heads}) + P(\text{all tails})]$

$$= 1 - \left[\frac{1}{2^n} + \frac{1}{2^n} \right]$$

$$= 1 - \frac{1}{2^{n-1}}$$

$$P(E_2) = P(\text{no head}) + P(\text{exactly one head})$$

$$= \frac{1}{2^n} + {}^nC_1 \cdot \frac{1}{2^n} = \frac{n+1}{2^n}$$

$$P(E_1 \cap E_2) = (\text{exactly one head and } (n-1) \text{ tail})$$

$$= {}^nC_1 \cdot \frac{1}{2} \cdot \frac{1}{2^{n-1}} = \frac{n}{2^n}$$

If E_1 and E_2 are independent, then

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$$\Rightarrow \frac{n}{2^n} = \left(1 - \frac{1}{2^{n-1}}\right) \left(\frac{n+1}{2^n}\right)$$

$$\Rightarrow n = \left(1 - \frac{1}{2^{n-1}}\right) (n+1)$$

$$\Rightarrow n = n+1 - \frac{n+1}{2^{n-1}}$$

$$\Rightarrow n+1 = 2^{n-1}$$

$$\Rightarrow n = 3$$

CONCEPT APPLICATION EXERCISE 14.3

1. A coin is tossed three times.
Event A : two heads appear
Event B : last should be head
Then identify whether events A and B are independent or not.
2. Events A and B are such that $P(A) = 1/2$, $P(B) = 7/12$, and $P(\text{not } A \text{ or not } B) = 1/4$. State whether A and B are independent?
3. Two cards are drawn one by one randomly from a pack of 52 cards. Then find the probability that both of them are king.
4. The probability of happening an event A in one trial is 0.4. Find the probability that the event A happens at least once in three independent trials.
5. In a bag there are 6 balls of which 3 are white and 3 are black. They are drawn successively with replacement. What is the chance that the colours are alternate?
6. In ten trials of an experiment, if the probability of getting '4 successes' is maximum, then find the probability of failure in each trial.
7. A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is $1/4$ and that of the woman's selection is $1/3$. What is the probability that none of them will be selected?
8. The probability that Krishna will be alive 10 years hence is $7/15$ and that Hari will be alive is $7/10$. What is the probability that both Krishna and Hari will be dead 10 years hence?
9. A binary number is made up of 8 digits. Suppose that the probability of an incorrect digit appearing is p and that the errors in different digits are independent of each other. Then find the probability of forming an incorrect number.
10. The probability of India winning a test match against West Indies is $1/2$. Assuming independence from match to match, find the probability that in a match series India's second win occurs at the third test.
11. Three persons A , B and C , in order, cut a pack of cards replacing them after each cut on the condition that the first who cuts a spade shall win the prize. Find their respective chances.
12. A bag contains a white and b black balls. Two players, A and B alternately draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. A begins the game. If the probability of A winning the game is three times that of B , then find the ratio $a : b$.

ANSWERS

1. Not independent 2. Not independent 3. $1/221$
 4. 0.784 5. $\frac{1}{32}$ 6. $3/5$ 7. $1/2$ 8. $4/25$
 9. $1 - (1 - p)^8$ 10. $1/4$ 11. $16/37, 12/37, 9/37$
 12. $a : b = 2 : 1$

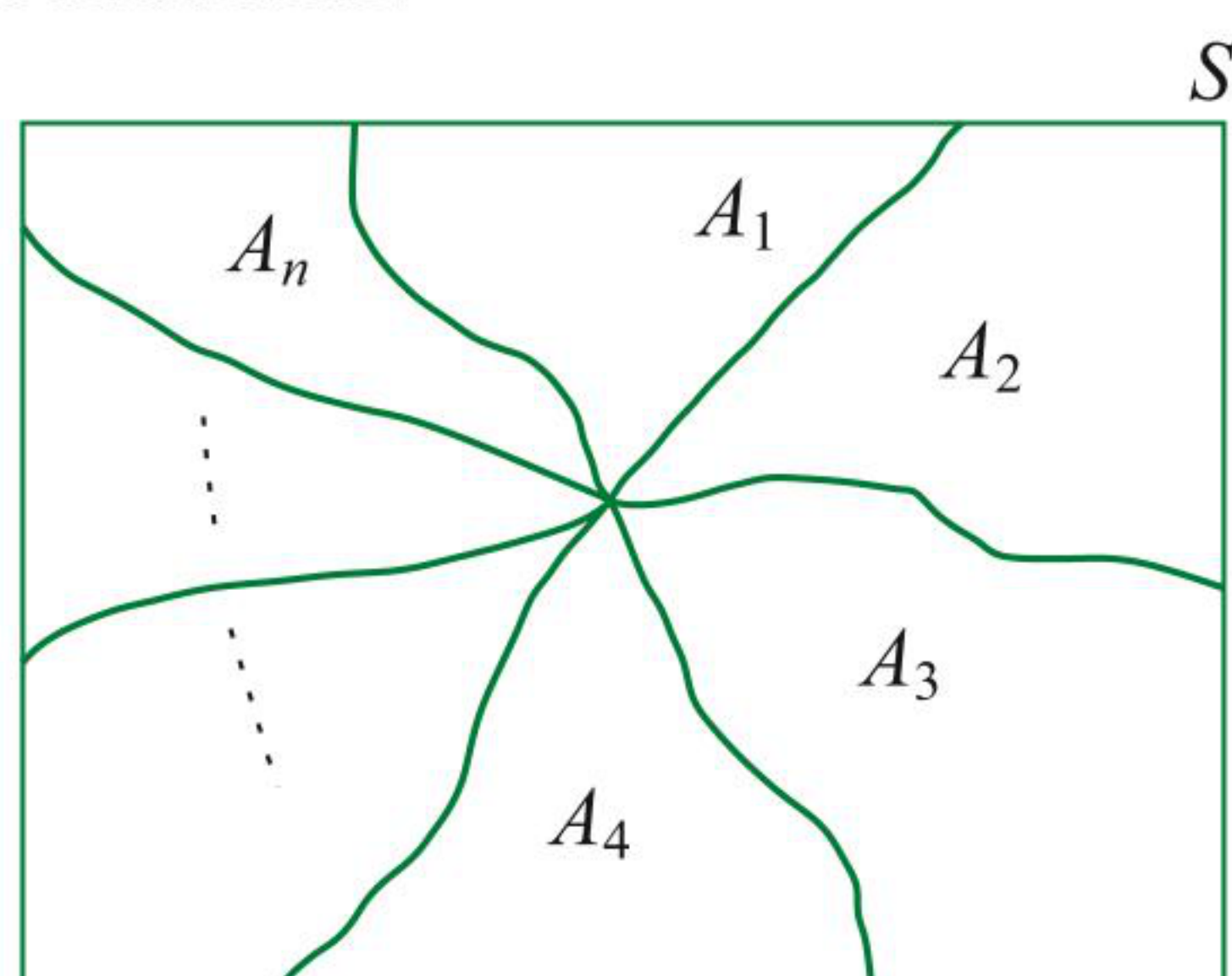
TOTAL PROBABILITY THEOREM

PARTITION OF A SAMPLE SPACE

A set of events $A_1, A_2, \dots, A_n$ is said to represent a partition of the sample space S if

- (a) $A_i \cap A_j = \phi, i \neq j, i, j = 1, 2, 3, \dots, n$
 (b) $A_1 \cup A_2 \cup \dots \cup A_n = S$
 (c) $P(A_i) > 0$ for all $i = 1, 2, \dots, n$.

In other words, the events $A_1, A_2, \dots, A_n$ represent a partition of the sample space S if they are pairwise disjoint, exhaustive and have non-zero probabilities.



Any non-empty events A and its complement A' form a partition of sample space since A and A' are mutually exclusive and exhaustive.

The partition of a sample space is not unique. There can be several partitions of the same sample space.

TOTAL PROBABILITY THEOREM

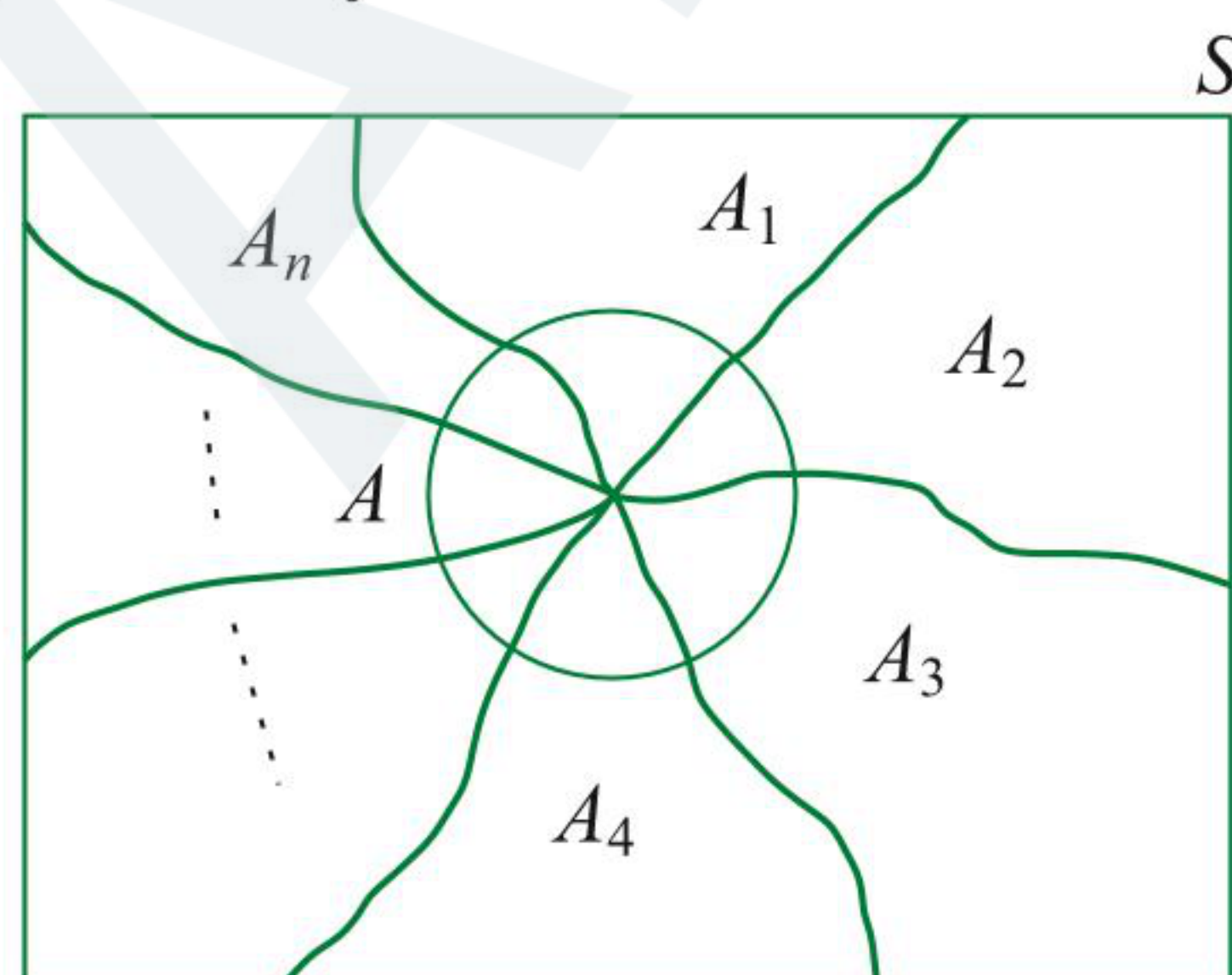
Let $\{A_1, A_2, \dots, A_n\}$ be a partition of the sample space S , and suppose that each of the events $A_1, A_2, \dots, A_n$ has non-zero probability of occurrence.

Let A be any event associated with S . Then

$$P(A) = P(A_1) P(A/A_1) + P(A_2) P(A/A_2) + \dots + P(A_n) P(A/A_n) \\ = \sum_{i=1}^n P(A_i) P(A/A_i)$$

This is called total probability theorem.

Proof:



From the figure,

$$A = A \cap S = A \cap (A_1 \cup A_2 \cup \dots \cup A_n) \\ = (A \cap A_1) \cup (A \cap A_2) \cup \dots \cup (A \cap A_n)$$

Clearly, $(A \cap A_1), (A \cap A_2), \dots, (A \cap A_n)$ are also disjoint events.

By addition theorem,

$$P(A) = P(A \cap A_1) + P(A \cap A_2) + \dots + P(A \cap A_n) \\ = P(A_1) P(A/A_1) + P(A_2) P(A/A_2) + \dots + P(A_n) P(A/A_n) \\ \text{(Using multiplication rule of probability)}$$

$$= \sum_{i=1}^n P(A_i) P(A/A_i)$$

ILLUSTRATION 14.25

'X' speaks truth in 60% and 'Y' in 50% of the cases. Find the probability that they contradict each other narrating the same incident.

Sol. Let event X be that X speaks the truth and event Y be that Y speaks the truth.

According to the question,

$$P(X) = 0.6 \text{ and } P(Y) = 0.5$$

$$\therefore P(X') = 0.4 \text{ and } P(Y') = 0.5$$

Let event C be "X and Y contradict each other narrating the same incident".

Clearly, event C occurs when one of X and Y speaks the truth and other does not.

$$\therefore P(C) = P(X \cap Y') + P(X' \cap Y) \\ = P(X) P(Y') + P(X') P(Y) \\ \text{(since } X \text{ and } Y \text{ are independent)} \\ = 0.6 \times 0.5 + 0.4 \times 0.5 \\ = 0.5$$

ILLUSTRATION 14.26

A person has undertaken a construction job. The probabilities are 0.80 that there will be a strike, 0.70 that the construction job will be completed on time if there is no strike, and 0.4 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

Sol. Let A be the event "the construction job will be completed on time" and B be the event "there will be a strike".

According to the question,

$$P(B) = 0.8$$

$$\therefore P(B') = P(\text{there will be no strike}) = 0.2$$

Also, $P(A/B') = P(\text{construction job will be completed on time if there is no strike})$

$$= 0.7$$

and $P(A/B) = P(\text{construction job will be completed on time if there is a strike})$

$$= 0.4$$

Using total probability theorem, we get

$$P(A) = P(B) P(A/B) + P(B') P(A/B') \\ = 0.8 \times 0.4 + 0.2 \times 0.7 \\ = 0.32 + 0.14 \\ = 0.46$$

ILLUSTRATION 14.27

A bag contains $n + 1$ coins. It is known that one of these coins shows heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $7/12$, then find the value of n .

Sol. Let E_1 denote an event when a coin with two heads is selected and E_2 an event when a fair coin is selected. Let A be the event when the toss results in head. Then,

$$P(E_1) = 1/(n + 1), P(E_2) = n/(n + 1), P(A/E_1) = 1,$$

$$\text{and } P(A/E_2) = 1/2.$$

Using total probability theorem, we have

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

$$\text{or } \frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$$

$$\text{or } 12 + 6n = 7n + 7$$

$$\text{or } n = 5$$

ILLUSTRATION 14.28

A lot contains 20 articles. The probability that the lot contains exactly two defective articles is 0.4 and the probability that it contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. Then find the probability that the testing procedure ends at the twelfth testing.

Sol. Let E_1 denote the event that the lot contains 2 defective articles and E_2 the event that the lot contains 3 defective articles. Suppose that A denotes the event that all the defective articles are found by the twelfth testing, then we have

$$P(E_1) = 0.4, P(E_2) = 0.6$$

$$\text{Now, } P(E_1) + P(E_2) = 0.4 + 0.6 = 1$$

Therefore there can be no other possibility. Now,

$$P(A/E_1) = \frac{{}^2C_1 {}^{18}C_{10}}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{11}{190}$$

(Here up to 11th draw, 1 defective and 10 non-defective articles are drawn and the second (i.e., last) defective article is drawn at twelfth draw.)

Also,

$$P(A/E_2) = \frac{{}^3C_2 {}^{17}C_9}}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{11}{228}$$

[Here up to 11th draw, 2 defective and 9 non-defective articles are drawn and the third (i.e., last) defective articles is drawn at the twelfth draw.]

Using total probability theorem, we have

$$\begin{aligned} P(A) &= P(E_1) P(A/E_1) + P(E_2) P(A/E_2) \\ &= 0.4 \times \frac{11}{190} + 0.6 \times \frac{11}{228} = \frac{99}{1990} \end{aligned}$$

ILLUSTRATION 14.29

Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B . Then one ball is drawn at random from urn B and placed in urn A . If one ball is now drawn at random from urn A , the probability that it is red.

Sol. First draw is from urn A , second draw is from urn B and third draw is again from urn A .

Let events A_R and A_B be drawing red ball and black ball, respectively, from bag A .

Also, let events B_R and B_B be drawing red ball and black ball, respectively, from bag B .

Case I:

$$A_R \rightarrow B_R \rightarrow A_R$$

Required probability,

$$\begin{aligned} P(A_R \cap B_R \cap A_R) &= P(A_R) P(B_R/A_R) P(A_R/(A_R \cap B_R)) \\ &= \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} \\ &= \frac{18}{110} \end{aligned}$$

Case II:

$$A_R \rightarrow B_B \rightarrow A_R$$

Required probability

$$\begin{aligned} P(A_R \cap B_B \cap A_R) &= P(A_R) P(B_B/A_R) P(A_R/(A_R \cap B_B)) \\ &= \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} \\ &= \frac{18}{110} \end{aligned}$$

Case III:

$$A_B \rightarrow B_R \rightarrow A_R$$

Required probability

$$\begin{aligned} P(A_B \cap B_R \cap A_R) &= P(A_B) P(B_R/A_B) P(A_R/(A_B \cap B_R)) \\ &= \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} \\ &= \frac{56}{550} \end{aligned}$$

Case IV:

$$A_B \rightarrow B_B \rightarrow A_R$$

Required probability

$$\begin{aligned} P(A_B \cap B_B \cap A_R) &= P(A_B) P(B_B/A_B) P(A_R/(A_B \cap B_B)) \\ &= \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} \\ &= \frac{84}{550} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total probability} &= \frac{18}{110} + \frac{18}{110} + \frac{56}{550} + \frac{84}{550} \\ &= \frac{90 + 90 + 56 + 84}{550} \\ &= \frac{320}{550} = \frac{32}{55} \end{aligned}$$

ILLUSTRATION 14.30

An urn contains 6 white and 4 black balls. A fair dice is rolled and that number of balls are chosen from the urn. Find the probability that the balls selected are white.

Sol. Let A_i denote the event that the number i appears on the dice and let E denote the event that only white balls are drawn.

$$\text{Then } P(A_i) = \frac{1}{6} \text{ for } i = 1, 2, \dots, 6$$

$$\text{and } P(E/A_i) = \frac{{}^6C_i}{{}^{10}C_i}, i = 1, 2, \dots, 6$$

Using total probability theorem, required probability,

$$\begin{aligned} P(E) &= \sum_{i=1}^6 P(E \cap A_i) \\ &= \sum_{i=1}^6 P(A_i)P(E/A_i) \\ &= \frac{1}{6} \left[\frac{6}{10} + \frac{15}{45} + \frac{20}{120} + \frac{15}{210} + \frac{6}{252} + \frac{1}{210} \right] = \frac{1}{5} \end{aligned}$$

ILLUSTRATION 14.31

Suppose families always have one, two, or three children, with probabilities $1/4$, $1/2$, and $1/4$, respectively. Assume everyone eventually gets married and has children, then find the probability of a couple having exactly four grandchildren.

Sol. Let the events be

A : exactly one child

B : exactly two children

C : exactly three children

E : couple has exactly 4 grandchildren

$$\therefore P(A) = \frac{1}{4}; P(B) = \frac{1}{2}; P(C) = \frac{1}{4}$$

$$\therefore P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$$

Now when couple having one child say D , then D must have four children, which is not possible.

When couple has two children, either each child has two children or one has three children and other has one child.

When couple has three children, any two of children have one child each and remaining has two children.

$$\begin{aligned} \therefore P(E) &= \frac{1}{4} \times 0 + \frac{1}{2} \left[\underbrace{\left(\frac{1}{2}\right)^2}_{(2,2)} + \underbrace{\frac{1}{4} \times \frac{1}{4} \times 2}_{(1,3) \text{ or } (3,1)} \right] + \frac{1}{4} \left[3 \underbrace{\left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{2}\right)}_{\substack{1 \quad 1 \quad 2}} \right] \\ &= \frac{1}{8} + \frac{1}{16} + \frac{3}{128} = \frac{27}{128} \end{aligned}$$

ILLUSTRATION 14.32

On a normal standard die one of the 21 dots from any one of the six faces is removed at random with each dot equally likely to be chosen. If the die is then rolled, then find the probability that the odd number appears.

Sol. E_1 : event that the dot is removed from an odd face;

$$P(E_1) = \frac{1+3+5}{21} = \frac{9}{21}$$

E_2 : dot is removed from the even face;

$$P(E_2) = \frac{2+4+6}{21} = \frac{12}{21}$$

E : odd number appears when die is thrown

Also when dot is removed from an odd numbered face, there are exactly 2 faces with odd number of dots, and when dot is removed from an even numbered face, there are exactly 4 faces with odd number of dots.

$\therefore$ Required probability

$$\begin{aligned} P(E) &= P(E \cap E_1) + P(E \cap E_2) \\ &= P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) \\ &= \left(\frac{9}{21}\right) \times \frac{2}{6} + \left(\frac{12}{21}\right) \times \frac{4}{6} = \frac{11}{21} \end{aligned}$$

CONCEPT APPLICATION EXERCISE 14.4

1. An urn contains 5 red and 5 black balls. A ball is drawn at random, its color is noted and is returned to the urn. Moreover, 2 additional balls of the color drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?
2. A bag contains 3 white, 3 black and 2 red balls. One by one, three balls are drawn without replacing them. Find the probability that the third ball is red.
3. Two-thirds of the students in a class are boys and the rest are girls. It is known that probability of a girl getting a first class is 0.25 and that of a boy is 0.28. Then find the probability that a student chosen at random will get a first class.
4. A number is selected at random from the first 25 natural numbers. If it is a composite number, then it is divided by 5. But if it is not a composite number, it is divided by 2. Find the probability that there will be no remainder in the division.
5. A real estate man has eight master keys to open several new homes. Only one master key will open any given house. If 40% of these homes are usually left unlocked, what is the probability that the real estate man can get into a specific home if he selects three master keys at random before leaving the office?
6. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same color as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white?
7. A box contains 12 red and 6 white balls. Balls are drawn from the bag one at a time without replacement. If in 6 draws, there are at least 4 white balls, find the probability that exactly one white ball is drawn in the next two draws. (Binomial coefficients can be left as such.)

ANSWERS

1. $1/2$ 2. $1/4$ 3. 0.27
 4. 0.2 5. $5/8$ 6. $\frac{m}{m+n}$
 7. $\frac{{}^{10}C_1 \times {}^2C_1}{{}^{12}C_2} \times \frac{{}^{12}C_2 \times {}^6C_4}{{}^{18}C_6} + \frac{{}^{11}C_1 \times {}^1C_1}{{}^{12}C_1} \times \frac{{}^{12}C_1 \times {}^6C_5}{{}^{18}C_6}$

BAYES' THEOREM

Let $A_1, A_2, \dots, A_n$ be n non-empty events which constitute a partition of sample space S . So, $A_1, A_2, \dots, A_n$ are pairwise disjoint and $A_1 \cup A_2 \cup \dots \cup A_n = S$. If A is any event of non-zero probability, then

$$P(A_i / A) = \frac{P(A_i) \cdot P(A / A_i)}{\sum_{j=1}^n P(A_j) \cdot P(A / A_j)}$$

Proof:

Using formula of conditional probability, we have

$$\begin{aligned} P(A_i / A) &= \frac{P(A_i \cap A)}{P(A)} \\ &= \frac{P(A_i)P(A / A_i)}{P(A)} \quad (\text{Using multiplication rule of probability}) \\ &= \frac{P(A_i) \cdot P(A / A_i)}{\sum_{j=1}^n P(A_j) \cdot P(A / A_j)} \quad (\text{Using total probability theorem}) \end{aligned}$$

In particular, if $P(A_1) = P(A_2) = \dots = P(A_n)$, then

$$P(A_i / A) = \frac{P(A / A_i)}{\sum_{j=1}^n P(A / A_j)}$$

Bayes' theorem is also called the formula for the probability of "causes". Since the A_i 's are a partition of the sample space S , one and only one of the events A_i 's occurs (i.e., one of the events A_i 's must occur and only one can occur). Hence, the above formula gives us the probability of a particular A_i (i.e., a "Cause"), given that the event A has occurred.

In Bayes' theorem, the events $A_1, A_2, \dots, A_n$ are called hypotheses. The probability $P(A_i)$ is called the priori probability of the hypothesis A_i . The conditional probability $P(A_i / A)$ is called a posteriori probability of the hypothesis A_i .

ILLUSTRATION 14.33

A pack of playing cards was found to contain only 51 cards. If the first 13 cards, which are examined, are all red, what is the probability that the missing card is black?

Sol. Let A_1 be the event that black card is lost, A_2 be the event that red card is lost, and let A denote occurrence of first 13 cards which are examined and are found to be all red. Then, we have to find $P(A_1 / A)$. We have $P(A_1) = P(A_2) = 1/2$. Also, $P(A / A_1) = {}^{26}C_{13} / {}^{51}C_{13}$ and $P(A / A_2) = {}^{25}C_{13} / {}^{51}C_{13}$. Then by Bayes's rule,

$$P(A_1 / A) = \frac{P(A_1)P(A / A_1)}{P(A_1)P(A / A_1) + P(A_2)P(A / A_2)}$$

$$\begin{aligned} &= \frac{\frac{1}{2} \frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{1}{2} \frac{{}^{26}C_{13}}{{}^{51}C_{13}} + \frac{1}{2} \frac{{}^{25}C_{13}}{{}^{51}C_{13}}} \\ &= \frac{{}^{26}C_{13}}{{}^{26}C_{13} + {}^{25}C_{13}} = \frac{2}{2+1} = \frac{2}{3} \end{aligned}$$

ILLUSTRATION 14.34

An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probability of accidents are 0.01, 0.03, and 0.15, respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Sol. Let E_1, E_2 , and E_3 be the respective events that the driver is a scooter driver, a car driver, and a truck driver.

Let A be the event that the person meets with an accident.

There are 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers.

Total number of drivers = 2000 + 4000 + 6000 = 12000

$$P(E_1) = P(\text{driver is a scooter driver}) = \frac{2000}{12000} = \frac{1}{6}$$

$$P(E_2) = P(\text{driver is a car driver}) = \frac{4000}{12000} = \frac{1}{3}$$

$$P(E_3) = P(\text{driver is a truck driver}) = \frac{6000}{12000} = \frac{1}{2}$$

$$\begin{aligned} P(A / E_1) &= P(\text{scooter driver met with an accident}) \\ &= 0.01 = \frac{1}{100} \end{aligned}$$

$$\begin{aligned} P(A / E_2) &= P(\text{car driver met with an accident}) \\ &= 0.03 = \frac{3}{100} \end{aligned}$$

$$\begin{aligned} P(A / E_3) &= P(\text{truck driver met with an accident}) \\ &= 0.15 = \frac{15}{100} \end{aligned}$$

The probability that the driver is a scooter driver, given that he met with an accident, is given by $P(E_1 / A)$.

By using Bayes' theorem, we obtain

$$\begin{aligned} P(E_1 / A) &= \frac{P(E_1) \cdot P(A / E_1)}{P(E_1) \cdot P(A / E_1) + P(E_2) \cdot P(A / E_2) + P(E_3) \cdot P(A / E_3)} \\ &= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}} \\ &= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{100} \left(\frac{1}{6} + 1 + \frac{15}{2} \right)} = \frac{1}{52} \end{aligned}$$

ILLUSTRATION 14.35

A laboratory blood test is 99% effective in detecting a certain disease, when it is in fact present. However, the test also yields a false positive result for 0.5% of the healthy person tested (that is, if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Sol. Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Since E_1 and E_2 are events complimentary to each other, we have

$$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$$

Let A be the event that the blood test result is positive.

$$\begin{aligned} P(A/E_1) &= P(\text{result is positive given the person has disease}) \\ &= 99\% = 0.99 \end{aligned}$$

$$P(A/E_2) = P(\text{result is positive given that the person has no disease}) = 0.5\% = 0.005.$$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1 | A)$.

By using Bayes' theorem, we obtain

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)} \\ &= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005} \\ &= \frac{0.00099}{0.005985} = \frac{22}{133} \end{aligned}$$

ILLUSTRATION 14.36

In an entrance test, there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then find the probability that he was guessing.

Sol. We define the following events:

A_1 : He knows the answer

A_2 : He does not know the answer

E : He gets the correct answer

Then, $P(A_1) = 9/10$, $P(A_2) = 1 - 9/10 = 1/10$,

$$P(E/A_1) = 1, P(E/A_2) = 1/4.$$

Therefore, the required probability is

$$\begin{aligned} P(A_2/E) &= \frac{P(A_2)P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)} \\ &= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{9}{10} \times 1 + \frac{1}{10} \times \frac{1}{4}} = \frac{1}{37} \end{aligned}$$

ILLUSTRATION 14.37

Each of the n urns contains 4 white and 6 black balls. The $(n+1)$ th urn contains 5 white and 5 black balls. One of the $n+1$ urns is chosen at random and two balls are drawn from it without replacement. Both the balls turn out to be black. If the probability that the $(n+1)$ th urn was chosen to draw the balls is $1/16$, then find the value of n .

Sol. Let E_1 denote the event that one of the first n urns is chosen and E_2 denote the event that $(n+1)$ th urn is selected. A denotes the event that two balls drawn are black. Then,

$$P(E_1) = n/(n+1), P(E_2) = 1/(n+1),$$

$$P(A/E_1) = {}^6C_2 / {}^{10}C_2 = 1/3$$

$$\text{and } P(A/E_2) = {}^5C_2 / {}^{10}C_2 = 2/9.$$

Using Bayes' theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$\text{or } \frac{1}{16} = \frac{\left(\frac{1}{n+1}\right)\left(\frac{2}{9}\right)}{\left(\frac{n}{n+1}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{n+1}\right)\left(\frac{2}{9}\right)} = \frac{2}{3n+2}$$

$$\text{or } n = 10$$

ILLUSTRATION 14.38

Die A has 4 red and 2 white faces, whereas die B has 2 red and 4 white faces. A coin is flipped once. If it shows a head, the game continues by throwing die A ; if it shows tail, then die B is to be used. If the probability that die A is used is $32/33$ when it is given that red turns up every time in first n throws, then find the value of n .

Sol. Let R be the event that a red face appears in each of the first n throws.

E_1 : Die A is used when head has already fallen

E_2 : Die B is used when tail has already fallen

$$\therefore P\left(\frac{R}{E_1}\right) = \left(\frac{2}{3}\right)^n \text{ and } P\left(\frac{R}{E_2}\right) = \left(\frac{1}{3}\right)^n$$

As per the given condition,

$$\frac{P(E_1)P(R/E_1)}{P(E_1)P(R/E_1) + P(E_2)P(R/E_2)} = \frac{32}{33}$$

$$\text{or } \frac{\frac{1}{2}\left(\frac{2}{3}\right)^n}{\frac{1}{2}\left(\frac{2}{3}\right)^n + \frac{1}{2}\left(\frac{1}{3}\right)^n} = \frac{32}{33}$$

$$\text{or } \frac{2^n}{2^n + 1} = \frac{32}{33}$$

$$\text{or } n = 5$$

ILLUSTRATION 14.39

A bag contains n balls out of which some balls are white. If probability that a bag contains exactly i white ball is proportional to i^2 . A ball is drawn at random from the bag and found to be white, then find the probability that bag contains exactly 2 white balls.

Sol. Let event A_i is 'bag contains exactly i white balls'. According to the question, $P(A_i) = ki^2$.

$$\therefore \sum_{i=1}^n ki^2 = 1$$

$$\therefore k = \frac{6}{n(n+1)(2n+1)}$$

$$\Rightarrow P(A_i) = \frac{6i^2}{n(n+1)(2n+1)}$$

Let event B is "white ball is drawn" then using total probability theorem,

$$\begin{aligned} P(B) &= \sum_{i=1}^n P(B / A_i) P(A_i) \\ &= \sum_{i=1}^n \left(\frac{i}{n} \cdot \frac{6i^2}{n(n+1)(2n+1)} \right) \\ &= \frac{6}{n^2(n+1)(2n+1)} \cdot \frac{n^2(n+1)^2}{4} \\ &= \frac{3(n+1)}{2(2n+1)} \end{aligned}$$

CONCEPT APPLICATION EXERCISE 14.5

1. A card from a pack of 52 cards is lost. From the remaining cards, two cards are drawn and are found to be spades. Find the probability that missing card is also a spade.
2. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. When one of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the two-headed coin?
3. Probability that A speaks truth is $4/5$. A coin is tossed. A reports that a head appears. Find the probability that actually there was head.
4. An urn contains five balls. Two balls are drawn and are found to be white. Find the probability that all the balls are white.
5. The chances of defective screws in three boxes A , B , C are $1/5$, $1/6$, $1/7$, respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Then find the probability that it came from box A .

6. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?
7. The probability that a particular day in the month of July is a rainy day is $3/4$. Two person whose credibility are $4/5$ and $2/3$, respectively, claim that 15th July was a rainy day. Find the probability that it was really a rainy day.

ANSWERS

- | | | | | |
|----------|----------|--------|--------|-----------|
| 1. 11/50 | 2. 4/9 | 3. 4/5 | 4. 1/2 | 5. 42/107 |
| 6. 14/29 | 7. 24/25 | | | |

BERNOULLI TRIAL AND BINOMIAL DISTRIBUTION

Bernoulli trial is also known as binomial trial where only two outcomes of a given experiment are possible. We call these outcomes "success" and "failure". The term "success" means result meeting specified conditions. If we flip a coin, only two outcomes are possible; head and tail. Hence, flipping a coin is a Bernoulli trial. If we consider head to be success then tail will be considered failure. If we roll a dice, then six outcomes are possible which are 1, 2, 3, 4, 5, 6 and hence, rolling a dice is not a Bernoulli trial. However, if dice is rolled to get number 6, then "getting number 6 on the dice" will be success and "not getting 6 on the dice" will be failure. Therefore, this experiment is Bernoulli trial. Here, the probabilities of success and failure in any trial remain unchanged. Some real-life examples of a Bernoulli trial are if a bulb is on or off, if a question is answered correctly or not, if a student has passed or failed, etc.

From the above discussion, we have the following characteristics of a Bernoulli trial:

- (1) It can have only two outcomes, that can be labelled as success and failure.
- (2) Probabilities of success and failure remain unchanged through each trial.
- (3) The trials are independent of each other.
- (4) Number of trials are fixed.

If p is the probability of success, then the probability of failure, $q = 1 - p$.

PROBABILITY INVOLVING BERNOULLI TRIALS

Consider the experiment in which we roll a dice five times and we want to find the probability of getting number 6 three times.

The probability of getting six (success), $p = \frac{1}{6}$

$\therefore$ Probability of not getting six (failure), $q = 1 - \frac{1}{6} = \frac{5}{6}$

If the sequence of the success and failure is p, p, p, q, q then its

probability is $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2$

But there are many sequences of success and failure. The number of sequences is equal to number of ways in which 3 successes and 2 failures can be arranged that is $\frac{5!}{3!2!} = {}^5C_3$. Here, 5C_3 is answer to “which 3 trials give success?”.

Hence, probability of getting 3 head is ${}^5C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2$.

This is fourth term from the start in the binomial expansion $\left(\frac{1}{6} + \frac{5}{6}\right)^5$ of $(p + q)^5$. In fact, each term in the expansion is the value of the probability of ‘ r ’ successes (getting six, r times) in the experiment of rolling dice five times. This is why we call it Binomial distribution.

Now, consider an experiment consisting of n Bernoulli trials, in which probability of success in each trial is p .

Then the probability of r successes (exactly r successes) is given by $(r + 1)$ th term in the expression of $(p + q)^n$.

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$\text{or } P(X = r) = {}^nC_{n-r} p^r q^{n-r},$$

where $p + q = 1$ and $r = 0, 1, 2, 3, \dots, n$.

In the experiment, probability of

- at least ‘ r ’ successes, $P(X \geq r) = \sum_{\lambda=r}^n {}^nC_{\lambda} p^{\lambda} q^{n-\lambda}$
- at most ‘ r ’ successes, $P(X \leq r) = \sum_{\lambda=0}^r {}^nC_{\lambda} p^{\lambda} q^{n-\lambda}$

ILLUSTRATION 14.40

A die is thrown 7 times. What is the chance that an odd number turns up (i) exactly 4 times, (ii) at least 4 times?

Sol. Probability of success is $3/6 = 1/2$.

$$\therefore p = \frac{1}{2} \text{ and } q = \frac{1}{2}$$

(i) For exactly 4 successes, the required probability is

$${}^7C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \frac{35}{128}$$

(ii) For at least 4 successes, the required probability is

$$\begin{aligned} & {}^7C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 + {}^7C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 \\ & \quad + {}^7C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^1 + {}^7C_7 \left(\frac{1}{2}\right)^7 \\ & = \frac{35}{128} + \frac{21}{128} + \frac{7}{128} + \frac{1}{128} \\ & = \frac{64}{128} = \frac{1}{2} \end{aligned}$$

ILLUSTRATION 14.41

Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Sol. A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed. Therefore,

$$p = P(\text{right-handed}) = \frac{9}{10}$$

$$q = P(\text{left-handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that at most six people are right-handed is given by

$$\sum_{r=0}^6 {}^{10}C_r p^r q^{10-r} = \sum_{r=0}^6 {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

ILLUSTRATION 14.42

An experiment succeeds twice as often as it fails. Then find the probability that in the next 6 trials, there will be at least 4 successes.

Sol. Let p be its probability of success and q be the failure. Then $p = 2q$. Also, $p + q = 1$. It gives $p = 2/3$ and $q = 1/3$.

$$\begin{aligned} P\{4 \text{ successes in 6 trials}\} &= {}^6C_4 p^4 q^2 \\ &= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 \end{aligned} \quad (1)$$

$$P\{5 \text{ successes in 6 trials}\} = {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) \quad (2)$$

$$P\{6 \text{ successes in 6 trials}\} = {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0 \quad (3)$$

Therefore, the probability that there are at least 4 successes is

$$\begin{aligned} & P\{\text{either 4 or 5 or 6 successes}\} \\ &= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + {}^6C_6 \left(\frac{2}{3}\right)^6 \\ &= \frac{496}{729} \end{aligned}$$

ILLUSTRATION 14.43

What is the probability of guessing correctly at least 8 out of 10 answers on a true-false examination?

Sol. $p(X \geq 8)$

$$\begin{aligned} &= {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \\ &= \left(\frac{1}{2}\right)^{10} [{}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0] \\ &= \left(\frac{1}{2}\right)^{10} [45 + 10 + 1] \\ &= \frac{56}{8 \times 2^7} = \frac{7}{128} \end{aligned}$$

ILLUSTRATION 14.44

A rifleman is firing at a distant target and hence, has only 10% chances of hitting it. Find the number of rounds, he must fire in order to have more than 50% chances of hitting it at least once.

Sol. Let a rifleman fires n number of rounds.

Probability of hitting the target, $p = \frac{1}{10}$.

$$\therefore \text{Probability of not hitting the target, } q = 1 - \frac{1}{10} = \frac{9}{10}.$$

$$\therefore \text{Probability of hitting the target at least once} = 1 - \left(\frac{9}{10}\right)^n$$

$$\text{Given that } 1 - \left(\frac{9}{10}\right)^n > \frac{1}{2}$$

$$\therefore \left(\frac{9}{10}\right)^n < \frac{1}{2}$$

So, the least value of n is 7.

ILLUSTRATION 14.45

A and B play a series of games which cannot be drawn and p , q are their respective chances of winning a single game. What is the chance that A wins m games before B wins n games?

Sol. For this to happen, A must win at least m out of the first $m + n - 1$ games. Therefore, the required probability is

$${}^{m+n-1}C_m p^m q^{n-1} + {}^{m+n-1}C_{m+1} p^{m+1} q^{n-2} + \dots + {}^{m+n-1}C_{m+n-1} p^{m+n-1}$$

CONCEPT APPLICATION EXERCISE 14.6

1. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then find the value of n .
2. A die is thrown 4 times. Find the probability of getting at most two 6.
3. The probability that a student is not a swimmer is $1/5$. Then find the probability that out of 5 students exactly 4 are swimmer.
4. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a “best of 3 games” or a “best of 5 games” match against B , which option should be chosen so that the probability of his winning the match is higher? (No game ends in a draw.)
5. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02 ..., 99 with replacement. An event E occurs if the only product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.

ANSWERS

1. $n = 14$
2. 0.984
3. $\left(\frac{4}{5}\right)^4$
4. First offer
5. $\frac{97}{(25)^4}$

Exercises

Single Correct Answer Type

- A man has 3 pairs of black socks and 2 pairs of brown socks kept together in a box. If he dressed hurriedly in the dark, the probability that after he has put on a black sock, he will then put on another black sock is
(1) $1/3$ (2) $2/3$ (3) $3/5$ (4) $2/15$
- There are 20 cards. Ten of these cards have the letter "P" printed on them and the other 10 have the letter "T" printed on them. If three cards are picked up at random and kept in the same order, the probability of making word IIT is
(1) $4/27$ (2) $5/38$ (3) $1/8$ (4) $9/80$
- One ticket is selected at random from 100 tickets numbered 00, 01, 02, ..., 98, 99. If x_1 and x_2 denotes the sum and product of the digits on the tickets, then $P(x_1 = 9/x_2 = 0)$ is equal to
(1) $2/19$ (2) $19/100$
(3) $1/50$ (4) none of these
- Let A and B be two events such that $P(A \cap B') = 0.20$, $P(A' \cap B) = 0.15$, $P(A' \cap B') = 0.1$, then $P(A/B)$ is equal to
(1) $11/14$ (2) $2/11$ (3) $2/7$ (4) $1/7$
- A father has 3 children with at least one boy. The probability that he has 2 boys and 1 girl is
(1) $1/4$ (2) $1/3$
(3) $2/3$ (4) None of these
- In a certain town, 40% of the people have brown hair, 25% have brown eyes, and 15% have both brown hair and brown eyes. If a person selected at random from the town has brown hair, the probability that he also has brown eyes is
(1) $1/5$ (2) $3/8$ (3) $1/3$ (4) $2/3$
- Let A and B are events of an experiment and $P(A) = 1/4$, $P(A \cup B) = 1/2$, then value of $P(B/A^c)$ is
(1) $2/3$ (2) $1/3$ (3) $5/6$ (4) $1/2$
- The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is
(1) 0.3 (2) 0.4 (3) 0.5 (4) 0.6
- A pair of numbers is picked up randomly (without replacement) from the set $\{1, 2, 3, 5, 7, 11, 12, 13, 17, 19\}$. The probability that the number 11 was picked given that the sum of the numbers was even is nearly
(1) 0.1 (2) 0.125 (3) 0.24 (4) 0.18
- All the jacks, queens, kings, and aces of a regular 52 cards deck are taken out. The 16 cards are thoroughly shuffled and my opponent, a person who always tells the truth, simultaneously draws two cards at random and says, "I hold at least one ace". The probability that he holds two aces is
(1) $2/8$ (2) $4/9$ (3) $2/3$ (4) $1/9$
- One ticket is selected at random from 100 tickets numbered 00, 01, 02, ..., 99. Suppose A and B are the sum and product of the digit found on the ticket, respectively. Then $P((A = 7)/(B = 0))$ is given by
(1) $2/13$ (2) $2/19$
(3) $1/50$ (4) none of these
- Two dices are rolled one after the other. The probability that the number on the first is smaller than the number on the second is
(1) $1/2$ (2) $7/18$ (3) $3/4$ (4) $5/12$
- Cards are drawn one-by-one at random from a well-shuffled pack of 52 playing cards until 2 aces are obtained from the first time. The probability that 18 draws are required for this is
(1) $3/34$ (2) $17/455$
(3) $561/15925$ (4) none of these
- A bag contains n white and n black balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each pair consists of one white and one black ball is
(1) $1/2^n C_n$ (2) $2n/2^n C_n$
(3) $2n/n!$ (4) $2n/(2n!)$
- A six-faced dice is so biased that it is twice as likely to show an even number as an odd number when thrown. It is thrown twice, the probability that the sum of two numbers thrown is even is
(1) $1/12$ (2) $1/6$ (3) $1/3$ (4) $5/9$
- A student appears for tests I, II, and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, and III are, respectively, p , q , and $1/2$. If the probability that the student is successful is $1/2$, then $p(1 + q) =$
(1) $1/2$ (2) 1 (3) $3/2$ (4) $3/4$
- A problem in mathematics is given to three students A , B , C and their respective probability of solving the problem is $1/2$, $1/3$, and $1/4$. Probability that the problem is solved is
(1) $3/4$ (2) $1/2$ (3) $2/3$ (4) $1/3$
- Let A, B, C be three mutually independent events. Consider the two statements S_1 and S_2 .
 S_1 : A and $B \cup C$ are independent.
 S_2 : A and $B \cap C$ are independent.
Then
(1) both S_1 and S_2 are true (2) only S_1 is true
(3) only S_2 is true (4) neither S_1 nor S_2 is true
- Three ships A , B , and C sail from England to India. If the ratio of their arriving safely are 2:5, 3:7, and 6:11, respectively, then the probability of all the ships for arriving safely is
(1) $18/595$ (2) $6/17$ (3) $3/10$ (4) $2/7$

20. Cards are drawn one by one without replacement from a pack of 52 cards. The probability that 10 cards will precede the first ace is
 (1) $241/1456$ (2) $164/4165$
 (3) $451/884$ (4) none of these
21. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is
 (1) $3/5$ (2) $1/5$ (3) $2/5$ (4) $4/5$
22. Let A and B be two events such that $P(\overline{A \cup B}) = 1/6$, $P(A \cap B) = 1/4$ and $P(\overline{A}) = 1/4$, where $\overline{A}$ stands for complement of event A . Then events A and B are
 (1) equally likely but not independent
 (2) equally likely and mutually exclusive
 (3) mutually exclusive and independent
 (4) independent but not equally likely
23. A class consists of 80 students, 25 of them are girls and 55 are boys. If 10 of them are rich and the remaining are poor and also 20 of them are intelligent, then the probability of selecting an intelligent rich girl is
 (1) $5/128$ (2) $25/128$
 (3) $5/512$ (4) none of these
24. Events A and C are independent. If the probabilities relating A , B , and C are $P(A) = 1/5$, $P(B) = 1/6$; $P(A \cap C) = 1/20$; $P(B \cup C) = 3/8$. Then
 (1) events B and C are independent
 (2) events B and C are mutually exclusive
 (3) events B and C are neither independent nor mutually exclusive
 (4) events B and C are equiprobable
25. Let A and B be two events. Suppose $P(A) = 0.4$, $P(B) = p$, and $P(A \cup B) = 0.7$. The value of p for which A and B are independent is
 (1) $1/3$ (2) $1/4$ (3) $1/2$ (4) $1/5$
26. A box contains 2 black, 4 white, and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept aside the first. This process is repeated till all the balls are drawn from the box. The probability that the balls drawn are in the sequence of 2 black, 4 white, and 3 red is
 (1) $1/1260$ (2) $1/7560$
 (3) $1/126$ (4) none of these
27. If any four numbers are selected and they are multiplied, then the probability that the last digit will be 1, 3, 5 or 7 is
 (1) $4/625$ (2) $18/625$
 (3) $16/625$ (4) none of these
28. If odds against solving a question by three students are 2:1, 5:2 and 5:3, respectively, then probability that the question is solved only by one student is
 (1) $31/56$ (2) $24/56$
 (3) $25/56$ (4) none of these
29. An unbiased coin is tossed 6 times. The probability that third head appears on the sixth trial is
 (1) $5/16$ (2) $5/32$ (3) $5/8$ (4) $5/64$
30. A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is
 (1) $3/16$ (2) $5/32$ (3) $3/16$ (4) $1/8$
31. Three critics review a book. Odds in favor of the book are 5:2, 4:3, and 3:4, respectively, for the three critics. The probability that majority are in favor of the book is
 (1) $35/49$ (2) $125/343$ (3) $164/343$ (4) $209/343$
32. A and B play a game of tennis. The situation of the game is as follows: if one scores two consecutive points after a deuce, he wins; if loss of a point is followed by win of a point, it is deuce. The chance of a server to win a point is $2/3$. The game is at deuce and A is serving. Probability that A will win the match is (serves are changed after each game)
 (1) $3/5$ (2) $2/5$ (3) $1/2$ (4) $4/5$
33. An unbiased cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is
 (1) $16/216$ (2) $50/216$
 (3) $60/216$ (4) none of these
34. A fair die is tossed repeatedly. A wins if it is 1 or 2 on two consecutive tosses and B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely is
 (1) $1/3$ (2) $5/21$ (3) $1/4$ (4) $2/5$
35. Whenever horses a , b , c race together, their respective probabilities of winning the race are 0.3, 0.5, and 0.2, respectively. If they race three times, the probability that the same horse wins all the three races, and the probability that a , b , c each wins one race are, respectively
 (1) $8/50, 9/50$ (2) $16/100, 3/100$
 (3) $12/50, 15/50$ (4) $10/50, 8/50$
36. A man alternately tosses a coin and throws a die beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is
 (1) $3/4$ (2) $1/2$
 (3) $1/3$ (4) none of these
37. If p is the probability that a man aged x will die in a year, then the probability that out of n men $A_1, A_2, \dots, A_n$ each aged x , A_1 will die in an year and be the first to die is
 (1) $1 - (1 - p)^n$ (2) $(1 - p)^n$
 (3) $1/n [1 - (1 - p)^n]$ (4) $1/n (1 - p)^n$
38. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better-ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up, respectively, is
 (1) $16/31$ (2) $1/2$
 (3) $17/31$ (4) none of these

39. A pair of unbiased dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is
 (1) $2/5$ (2) $3/5$
 (3) $4/5$ (4) none of these
40. A fair coin is tossed 10 times. Then the probability that two heads do not occur consecutively is
 (1) $7/64$ (2) $1/8$ (3) $9/16$ (4) $9/64$
41. A die is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number 4 times, then probability of getting even number exactly once is
 (1) $1/6$ (2) $1/9$ (3) $5/36$ (4) $7/128$
42. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is
 (1) $8/9$ (2) $8/729$ (3) $8/243$ (4) $1/729$
43. The probability that a bulb produced by a factory will fuse after 150 days if used is 0.05. What is the probability that out of 5 such bulbs none will fuse after 150 days of use?
 (1) $1 - (19/20)^5$ (2) $(19/20)^5$
 (3) $(3/4)^5$ (4) $90(1/4)^5$
44. A box contains tickets numbered from 1 to 20. Three tickets are drawn from the box with replacement. The probability that the largest number on the tickets is 7 is
 (1) $2/19$ (2) $7/20$
 (3) $1 - (7/20)^3$ (4) none of these
45. Two players toss 4 coins each. The probability that they both obtain the same number of heads is
 (1) $5/256$ (2) $1/16$
 (3) $35/128$ (4) none of these
46. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tails is
 (1) $\frac{(2n!)}{(n!)^2} \left(\frac{1}{2}\right)^{2n}$ (2) $1 - \frac{(2n!)}{(n!)^2}$
 (3) $1 - \frac{(2n!)}{(n!)^2} \frac{1}{4^n}$ (4) none of these
47. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is
 (1) $5/64$ (2) $27/32$ (3) $5/32$ (4) $1/2$
48. In a game a coin is tossed $2n + m$ times and a player wins if he does not get any two consecutive outcomes same for at least $2n$ times in a row. The probability that player wins the game is
 (1) $\frac{m+2}{2^{2n}+1}$ (2) $\frac{2n+2}{2^{2n}}$ (3) $\frac{2n+2}{2^{2n+1}}$ (4) $\frac{m+2}{2^{2n}}$
49. If A and B each toss three coins. The probability that both get the same number of heads is
 (1) $1/9$ (2) $3/16$ (3) $5/16$ (4) $3/8$
50. A fair coin is tossed 100 times. The probability of getting tails 1, 3, ..., 49 times is
 (1) $1/2$ (2) $1/4$ (3) $1/8$ (4) $1/16$
51. A fair die is thrown 20 times. The probability that on the 10th throw, the fourth six appears is
 (1) ${}^{20}C_{10} \times 5^6/6^{20}$ (2) $120 \times 5^7/6^{10}$
 (3) $84 \times 5^6/6^{10}$ (4) none of these
52. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing a single event is
 (1) 0.56 (2) 0.54 (3) 0.38 (4) 0.94
53. The probability that a teacher will give a surprise test during any class meeting is $\frac{1}{5}$. If a student is absent twice, then the probability that the student will miss at least one test is
 (1) $\frac{4}{5}$ (2) $\frac{2}{5}$ (3) $\frac{7}{25}$ (4) $\frac{9}{25}$
54. There are two urns A and B . Urn A contains 5 red, 3 blue, and 2 white balls, urn B contains 4 red, 3 blue, and 3 white balls. An urn is chosen at random and a ball is drawn. Probability that the ball drawn is red is
 (1) $9/10$ (2) $1/2$ (3) $11/20$ (4) $9/20$
55. A bag contains 20 coins. If the probability that the bag contains exactly 4 biased coin is $1/3$ and that of exactly 5 biased coin is $2/3$, then the probability that all the biased coin are sorted out from the bag in exactly 10 draws is
 (1) $\frac{5}{10} \frac{{}^{16}C_6}{{}^{20}C_9} + \frac{1}{11} \frac{{}^{15}C_5}{{}^{20}C_9}$ (2) $\frac{2}{33} \left[\frac{{}^{16}C_6 + 5 {}^{15}C_5}{{}^{20}C_9} \right]$
 (3) $\frac{5}{33} \frac{{}^{16}C_7}{{}^{20}C_9} + \frac{1}{11} \frac{{}^{15}C_6}{{}^{20}C_9}$ (4) none of these
56. A bag contains 3 red and 3 green balls and a person draws out 3 at random. He then drops 3 blue balls into the bag and again draws out 3 at random. The chance that the 3 later balls being all of different colors is
 (1) 15% (2) 20% (3) 27% (4) 40%
57. A bag contains 20 coins. If the probability that bag contains exactly 4 biased coin is $1/3$ and that of exactly 5 biased coin is $2/3$, then the probability that all the biased coin are sorted out from the bag in exactly 10 draws is
 (1) $\frac{5}{33} \frac{{}^{16}C_6}{{}^{20}C_9} + \frac{1}{11} \frac{{}^{15}C_5}{{}^{20}C_9}$ (2) $\frac{2}{33} \left[\frac{{}^{16}C_6 + 5 {}^{15}C_5}{{}^{20}C_9} \right]$
 (3) $\frac{5}{33} \frac{{}^{16}C_7}{{}^{20}C_9} + \frac{1}{11} \frac{{}^{15}C_6}{{}^{20}C_9}$ (4) none of these
58. An urn contains 3 red balls and n white balls. Mr. A draws two balls together from the urn. The probability that they have the same color is $1/2$. Mr. B draws one balls form the urn, notes its color and replaces it. He then draws a second ball from the urn and finds that both balls have the same color is $5/8$. The possible value of n is
 (1) 9 (2) 6 (3) 5 (4) 1

59. A student can solve 2 out of 4 problems of mathematics, 3 out of 5 problem of physics, and 4 out of 5 problems of chemistry. There are equal number of books of math, physics, and chemistry in his shelf. He selects one book randomly and attempts 10 problems from it. If he solves the first problem, then the probability that he will be able to solve the second problem is
 (1) $2/3$ (2) $25/38$ (3) $13/21$ (4) $14/23$
60. An event X can take place in conjunction with any one of the mutually exclusive and exhaustive events A , B and C . If A , B , C are equiprobable and the probability of X is $5/12$, and the probability of X taking place when A has happened is $3/8$, while it is $1/4$ when B has taken place, then the probability of X taking place in conjunction with C is
 (1) $5/8$ (2) $3/8$
 (3) $5/24$ (4) none of these
61. An artillery target may be either at point I with probability $8/9$ or at point II with probability $1/9$. We have 55 shells, each of which can be fired either at point I or II. Each shell may hit the target, independent of the other shells, with probability $1/2$. Maximum number of shells must be fired at point I to have maximum probability is
 (1) 20 (2) 25 (3) 29 (4) 35
62. A bag contains some white and some black balls, all combinations of balls being equally likely. The total number of balls in the bag is 10. If three balls are drawn at random without replacement and all of them are found to be black, the probability that the bag contains 1 white and 9 black balls is
 (1) $14/55$ (2) $12/55$ (3) $2/11$ (4) $8/55$
63. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is
 (1) $1/17$ (2) $12/17$ (3) $17/30$ (4) $3/5$
64. A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighborhood are sick with the flu, denoted by F , while 10% are sick with the measles, denoted by M . A well-known symptom of measles is a rash, denoted by R . The probability of having a rash for a child sick with the measles is 0.95. However, occasionally children with the flu also develop a rash, with conditional probability 0.08. Upon examination the child, the doctor finds a rash. Then what is the probability that the child has the measles?
 (1) $91/165$ (2) $90/163$ (3) $82/161$ (4) $95/167$
65. On a Saturday night, 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a Saturday night smashed into a tree, the probability that the driver was under the influence of alcohol is
 (1) $3/7$ (2) $4/7$ (3) $5/7$ (4) $6/7$
66. A purse contains 2 six-sided dice. One is a normal fair die, while the other has two 1's, two 3's, and two 5's. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up is
 (1) $1/7$ (2) $1/4$ (3) $1/6$ (4) $1/24$
67. There are 3 bags which are known to contain 2 white and 3 black, 4 white and 1 black, and 3 white and 7 black balls, respectively. A ball is drawn at random from one of the bags and found to be a black ball. Then the probability that it was drawn from the bag containing the most black ball is
 (1) $7/15$ (2) $5/19$
 (3) $3/4$ (4) None of these
68. A hat contains a number of cards with 30% white on both sides, 50% black on one side and white on the other side, 20% black on both sides. The cards are mixed up, and a single card is drawn at random and placed on the table. Its upper side shows up black. The probability that its other side is also black is
 (1) $2/9$ (2) $4/9$ (3) $2/3$ (4) $2/7$

Multiple Correct Answers Type

- If A and B are two independent events such that $P(A) = 1/2$ and $P(B) = 1/5$, then
 (1) $P(A \cup B) = 3/5$ (2) $P(A/B) = 1/4$
 (3) $P(A/A \cup B) = 5/6$ (4) $P(A \cap B/\bar{A} \cup \bar{B}) = 0$
- Let A and B be two events such that $P(A \cap B') = 0.20$, $P(A' \cap B) = 0.15$, and $P(A \text{ and } B \text{ both fail}) = 0.10$. Then
 (1) $P(A/B) = 2/7$ (2) $P(A) = 0.3$
 (3) $P(A \cup B) = 0.55$ (4) $P(A/B) = 1/2$
- The probability that a married man watches a certain TV show is 0.4 and the probability that a married woman watches the show is 0.5. The probability that a man watches the show, given that his wife does, is 0.7. Then
 (1) the probability that married couple watches the show is 0.35
 (2) the probability that a wife watches the show given that her husband does is $7/8$
 (3) the probability that at least one person of a married couple will watch the show is 0.55
 (4) none of these
- A and B are two events defined as follows:
 A : It rains today with $P(A) = 40\%$
 B : It rains tomorrow with $P(B) = 50\%$
 Also, $P(\text{it rains today and tomorrow}) = 30\%$
 Also, $E_1: P((A \cap B)/(A \cup B))$ and $E_2: P(\{(A \cap \bar{B}) \text{ or } (B \cup \bar{A})\}/(A \cup B))$. Then which of the following is/are true?
 (1) A and B are independent
 (2) $P(A/B) < P(B/A)$
 (3) E_1 and E_2 are equiprobable
 (4) $P(A/(A \cup B)) = P(B/(A \cup B))$

5. Two numbers are randomly selected and multiplied. Consider two events E_1 and E_2 defined as
 E_1 : Their product is divisible by 5
 E_2 : Unit's places in their product is 5
 Which of the following statement is/are correct?
 (1) E_1 is twice as likely to occur as E_2
 (2) E_1 and E_2 are disjoint
 (3) $P(E_2/E_1) = 1/4$
 (4) $P(E_1/E_2) = 1$
6. The probability that a 50-year-old man will be alive at 60 is 0.83 and the probability that a 45-year-old woman will be alive at 55 is 0.87. Then
 (1) the probability that both will be alive is 0.7221
 (2) at least one of them will alive is 0.9779
 (3) at least one of them will alive is 0.8230
 (4) the probability that both will be alive is 0.6320
7. Which of the following statement is/are correct?
 (1) Three coins are tossed once. At least two of them must land the same way. No matter whether they land heads or tails, the third coin is equally likely to land either the same way or oppositely. So, the chance that all the three coins land the same way is $1/2$.
 (2) Let $0 < P(B) < 1$ and $P(A/B) = P(A/B^C)$. Then A and B are independent.
 (3) Suppose an urn contains "w" white and "b" black balls and a ball is drawn from it and is replaced along with "d" additional balls of the same color. Now a second ball is drawn from it. The probability that the second drawn ball is white is independent of the value of "d".
 (4) A, B, C simultaneously satisfy

$$P(ABC) = P(A)P(B)P(C)$$

$$P(ABC^C) = P(A)P(B)P(C^C)$$

$$P(A^C B C) = P(A^C)P(B)P(C)$$

$$P(A^C B^C C) = P(A^C)P(B^C)P(C)$$

$$P(A^C B^C C^C) = P(A^C)P(B^C)P(C^C)$$

 Then A, B, C are independent.
8. A bag initially contains 1 red and 2 blue balls. An experiment consisting of selecting a ball at random, noting its color and replacing it together with an additional ball of the same color. If three such trials are made, then
 (1) probability that at least one blue ball is drawn is 0.9
 (2) probability that exactly one blue ball is drawn is 0.2
 (3) probability that all the drawn balls are red given that all the drawn balls are of same color is 0.2
 (4) probability that at least one red ball is drawn is 0.6
9. $P(A) = 3/8$; $P(B) = 1/2$; $P(A \cup B) = 5/8$, which of the following do/does hold good?
 (1) $P(A^C/B) = 2P(A/B^C)$ (2) $P(B) = P(A/B)$
 (3) $15P(A^C/B^C) = 8P(B/A^C)$ (4) $P(A/B^C) = (A \cap B)$
10. In a precision bombing attack, there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. The number of bombs which should be dropped to give a 99% chance or better of completely destroying the target can be
 (1) 12 (2) 11 (3) 10 (4) 13
11. If A and B are two events, then which one of the following is/are always true?
 (1) $P(A \cap B) \geq P(A) + P(B) - 1$
 (2) $P(A \cap B) \leq P(A)$
 (3) $P(A' \cap B') \geq P(A') + P(B') - 1$
 (4) $P(A \cap B) = P(A)P(B)$
12. If A and B are two independent events such that $P(A) = 1/2$, $P(B) = 1/5$, then
 (1) $P(A/B) = 1/2$ (2) $P\left(\frac{A}{A \cup B}\right) = \frac{5}{6}$
 (3) $P\left(\frac{A \cap B}{A' \cup B'}\right) = 0$ (4) none of these
13. If A and B are two independent events such that $P(\bar{A} \cap B) = 2/15$ and $P(A \cap \bar{B}) = 1/6$, then $P(B)$ is
 (1) $1/5$ (2) $1/6$ (3) $4/5$ (4) $5/6$
14. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is $1/5$. The probability that bus B will be late is $7/25$. The probability that the bus B is late given that bus A is late is $9/10$. Then,
 (1) probability that neither bus will be late on a particular day is $7/10$
 (2) probability that bus A is late given that bus B is late is $18/28$
 (3) probability that at least one bus is late is $3/10$
 (4) probability that at least one bus is in time is $4/5$
15. A fair coin is tossed 99 times. Let X be the number of times heads occurs. Then $P(X = r)$ is maximum when r is
 (1) 49 (2) 52 (3) 51 (4) 50
16. If the probability of choosing an integer "k" out of $2m$ integers $1, 2, 3, \dots, 2m$ is inversely proportional to k^4 ($1 \leq k \leq m$). If x_1 is the probability that chosen number is odd and x_2 is the probability that chosen number is even, then
 (1) $x_1 > 1/2$ (2) $x_1 > 2/3$
 (3) $x_2 < 1/2$ (4) $x_2 < 2/3$
17. A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B and C are defined as follows:
 A = (first bulb is defective)
 B = (second bulb is non-defective)
 C = (two bulbs are both defective or both non-defective)
 Then
 (1) A and B are independent
 (2) B and C are independent
 (3) A and C are independent
 (4) A, B and C are pairwise independent

Linked Comprehension Type

For Problems 1–3

In a class of 10 students, probability of exactly i students passing an examination is directly proportional to i^2 . Then answer the following questions:

- The probability that exactly 5 students passing an examination is
(1) $1/11$ (2) $5/77$ (3) $25/77$ (4) $10/77$
- If a student is selected at random, then the probability that he has passed the examination is
(1) $1/7$ (2) $11/35$
(3) $11/14$ (4) none of these
- If a students selected at random is found to have passed the examination, then the probability that he was the only student who has passed the examination is
(1) $1/3025$ (2) $1/605$ (3) $1/275$ (4) $1/121$

For Problems 4–6

In an objective paper, there are two sections of 10 questions each. For “section 1”, each question has 5 options and only one option is correct and “section 2” has 4 options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in “section 1” is 1 and in “section 2” is 3. (There is no negative marking.)

- If a candidate attempts only two questions by guessing, one from “section 1” and one from “section 2”, the probability that he scores in both questions is
(1) $74/75$ (2) $1/25$ (3) $1/15$ (4) $1/75$
- If a candidate in total attempts 4 questions all by guessing, then the probability of scoring 10 marks is
(1) $1/15(1/15)^3$ (2) $4/5(1/15)^3$
(3) $1/5(14/15)^3$ (4) none of these
- The probability of getting a score less than 40 by answering all the questions by guessing in this paper is
(1) $(1/75)^{10}$ (2) $1 - (1/75)^{10}$
(3) $(74/75)^{10}$ (4) none of these

For Problems 7–9

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day, and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours, and 4 hours per day with probabilities 0.1, 0.2, and 0.7, respectively.

- The chance she will be successful is
(1) 0.28 (2) 0.38 (3) 0.48 (4) 0.58
- Given that she is successful, the chance that she studied for 4 hours is
(1) $6/12$ (2) $7/12$ (3) $8/12$ (4) $9/12$
- Given that she does not achieve success, the chance she studied for 4 hour is
(1) $18/26$ (2) $19/26$ (3) $20/26$ (4) $21/26$

For Problems 10–12

Let S and T are two events defined on a sample space with probabilities

$$P(S) = 0.5, P(T) = 0.69, P(S/T) = 0.5$$

- Events S and T are
(1) mutually exclusive
(2) independent
(3) mutually exclusive and independent
(4) neither mutually exclusive nor independent
- The value of $P(S \text{ and } T)$ is
(1) 0.3450 (2) 0.2500 (3) 0.6900 (4) 0.350
- The value of $P(S \text{ or } T)$ is
(1) 0.6900 (2) 1.19 (3) 0.8450 (4) 0

For Problems 13–15

An amoeba either splits into two or remains the same or eventually dies out immediately after completion of every second with probabilities, respectively, $1/2$, $1/4$, and $1/4$. Let the initial amoeba be called as mother amoeba and after every second, the amoeba, if it is distinct from the previous one, be called as 2nd, 3rd, ... generations.

- The probability that immediately after completion of 2 s all the amoeba population dies out is
(1) $9/32$ (2) $11/32$ (3) $1/2$ (4) $3/32$
- The probability that after 2 s exactly 4 amoeba are alive is
(1) $1/16$ (2) $1/8$ (3) $1/4$ (4) $1/2$
- The probability that amoeba population will be maximum after completion of 3 s is
(1) $1/2^7$ (2) $1/2^6$
(3) $1/2^8$ (4) none of these

For Problems 16–18

Two fair dice are rolled. Let $P(A_i) > 0$ denote the event that the sum of the faces of the dice is divisible by i .

- Which one of the following events is most probable?
(1) A_3 (2) A_4 (3) A_5 (4) A_6
- For which one of the following pairs (i, j) are the events A_i and A_j independent?
(1) (3, 4) (2) (4, 6) (3) (2, 3) (4) (4, 2)
- The number of all possible ordered pairs (i, j) for which the events A_i and A_j are independent is
(1) 6 (2) 12 (3) 13 (4) 25

For Problems 19–21

A player tosses a coin and scores one point for every head and two points for every tail that turns up. He plays on until his score reaches or passes n . P_n denotes the probability of getting a score of exactly n .

- The value of P_n is equal to
(1) $(1/2)[P_{n-1} + P_{n-2}]$ (2) $(1/2)[2P_{n-1} + P_{n-2}]$
(3) $(1/2)[P_{n-1} + 2P_{n-2}]$ (4) none of these
- The value of $P_n + (1/2)P_{n-1}$ is equal to
(1) $1/2$ (2) $2/3$
(3) 1 (4) none of these

21. Which of the following is not true?
- (1) $P_{100} > 2/3$

(2) $P_{101} < 2/3$

(3) $P_{100}, P_{101} > 2/3$

(4) none of these

Matrix Match Type

1. An urn contains four black and eight white balls. Three balls are drawn from the urn without replacement. Three events are defined on this experiment.
- A: Exactly one black ball is drawn.
B: All balls are drawn are of the same color.
C: Third drawn ball is black.
- Match the entries of List I with none, one or more entries of List II.

List I	List II
a. The events A and B are	p. mutually exclusive
b. The events B and C are	q. independent
c. The events C and A are	r. neither independent nor mutually exclusive
d. The events A, B and C are	s. exhaustive

2. Match the following lists:

List I	List II
a. If the probability of getting at least one head is at least 0.8 is n trials, then value of n can be	p. 2
b. One mapping is selected at random from all mappings of the set $s = \{1, 2, 3, \dots, n\}$ into itself. If the probability that the mapping being one-one is $3/32$, then find the value of n is	q. 3
c. If m is selected at random from set $\{1, 2, \dots, 10\}$ and the probability that the quadratic equation $2x^2 + 2mx + m + 1 = 0$ has real roots is k , then value of $5k$ is more than	r. 4
d. A man firing at a distant target as 20% chance of hitting the target in one shoot. If P be the probability of hitting the target in " n " attempts, where $20P^2 - 13P + 2 \leq 0$, then the ratio of maximum and minimum value of n is less than	s. 5

3. Match the following lists:

List I	List II
a. The probability of a bomb hitting a bridge is $1/2$. Two direct hits are needed to destroy it. The number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 can be	p. 4
b. A bag contains 2 red, 3 white, and 5 black balls, a ball is drawn its color is noted and replaced. The number of times, a ball can be drawn so that the probability of getting a red ball for the first time is at least $1/2$	q. 6
c. A drawer contains a mixture of red socks and blue socks, at most 17 in all. It so happens that when two socks are selected randomly without replacement, there is a probability of exactly $1/2$ that both are red or both are blue. Then number of red socks in the drawer can be	r. 7

d. There are two red, two blue, two white, and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer without replacement, the probability that they are of the same color is $1/5$, then the number of green socks are	s. 10
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4. An urn contains r red balls and b black balls. Now, match the following lists:

List I	List II
a. If the probability of getting two red balls in first two draws (without replacement) is $1/2$, then value of r can be	p. 10
b. If the probability of getting two red balls in first two draws (without replacement) is $1/2$ and b is an even number, then r can be	q. 3
c. If the probability of getting exactly two red balls in four draws (with replacement) from the urn is $3/8$ and $b = 10$, then r can be	r. 8
d. If the probability of getting exactly n red balls in $2n$ draw (with replacement) is equal to probability of getting exactly n black balls in $2n$ draws (with replacement), then the ratio r/b can be	s. 2

5. Let A and B are two independent events such that $P(A) = 1/3$ and $P(B) = 1/4$. Then match the following lists:

List I	List II
a. $P(A \cup B)$ is equal to	p. $1/12$
b. $P(A/A \cup B)$ is equal to	q. $1/2$
c. $P(B/A' \cap B')$ is equal to	r. $2/3$
d. $P(A'/B)$ is equal to	s. 0

Codes

- a

b

c

d
- (1)

q

s

s

r
- (2)

q

r

s

r
- (3)

q

s

r

p
- (4)

r

s

p

q

6. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random from the bag at random without replacement.

List I	List II
a. Probability that all the four balls are black is equal to	p. $14/33$
b. If the bag contains 10 black and 2 white balls, then the probability that all four balls are black is equal to	q. $1/3$
c. If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	r. $70/429$
d. Probability that two balls are black and two are white is	s. $13/165$

Codes

	a	b	c	d
(1)	q	s	s	r
(2)	r	s	q	p
(3)	q	s	r	p
(4)	q	p	r	q

Numerical Value Type

- If two loaded dice each have the property that 2 or 4 is three times as likely to appear as 1, 3, 5, or 6 on each roll. When two such dice are rolled, the probability of obtaining a total of 7 is p , then the value of $[1/p]$ is, where $[x]$ represents the greatest integer less than or equal to x .
- An urn contains three red balls and n white balls. Mr. A draws two balls together from the urn. The probability that they have the same color is $1/2$. Mr. B draws one balls from the urn, notes its color and replaces it. He then draws a second ball from the urn and finds that both balls have the same color is $5/8$. The possible value of n is _____.
- Suppose A and B are two events with $P(A) = 0.5$ and $P(A \cup B) = 0.8$. Let $P(B) = p$ if A and B are mutually exclusive and $P(B) = q$ if A and B are independent events, then the value of q/p is _____.
- Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players the better ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up respectively is p , then the value of $[2/p]$ is, where $[.]$ represents the greatest integer function, _____.
- If A and B are two events such that $P(A) = 0.6$ and $P(B) = 0.8$, if the greatest value that $P(A/B)$ can have is p , then the value of $8p$ is _____.
- A die is thrown three times. The chance that the highest number shown on the die is 4 is p , then the value of $[1/p]$ is where $[.]$ represents greatest integer function is _____.
- Two cards are drawn from a well shuffled pack of 52 cards. The probability that one is a heart card and the other is a king is p , then the value of $104p$ is _____.
- A fair coin is flipped n times. Let E be the event "a head is obtained on the first flip", and let F_k be the event "exactly k heads are obtained". Then the value of n/k for which E and F_k are independent is _____.
- An unbiased normal coin is tossed n times. Let E_1 : event that both heads and tails are present in n tosses. E_2 : event that the coin shows up heads at most once. The value of n for which E_1 and E_2 are independent is _____.
- In a knockout tournament 2^n equally skilled players, $S_1, S_2, \dots, S_{2^n}$, are participating. In each round, players are divided in pair at random and winner from each pair moves in the next round. If S_2 reaches the semi-final, then the probability that S_1 wins the tournament is $1/84$. The value of n equals _____.

Archives

JEE ADVANCED

Single Correct Answer Type

- A signal which can be green or red with probability $\frac{2}{3}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B . The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is
 (1) $\frac{3}{5}$ (2) $\frac{6}{7}$ (3) $\frac{20}{23}$ (4) $\frac{9}{20}$
 (IIT-JEE 2010)
- Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is
 (1) $\frac{235}{256}$ (2) $\frac{21}{256}$ (3) $\frac{3}{256}$ (4) $\frac{253}{256}$
 (JEE Advanced 2013)

- A computer producing factory has only two plants T_1 and T_2 . Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that $P(\text{computer turns out to be defective given that it is produced in plant } T_1) = 10 P(\text{computer turns out to be defective given that it is produced in plant } T_2)$, where $P(E)$ denotes the probability of an event E . A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is
 (1) $\frac{36}{73}$ (2) $\frac{47}{79}$
 (3) $\frac{78}{96}$ (4) $\frac{75}{83}$
 (JEE Advanced 2016)
- Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively.

Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently. Then the probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta = 0$ are real and equal, is

- (1) $\frac{40}{81}$ (2) $\frac{20}{81}$
 (3) $\frac{1}{2}$ (4) $\frac{1}{4}$ (JEE Advanced 2020)

5. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1 , and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

- (1) $\frac{1}{5}$ (2) $\frac{3}{5}$
 (3) $\frac{1}{2}$ (4) $\frac{2}{5}$ (JEE Advanced 2021)

Multiple Correct Answers Type

1. Let E and F be two independent events. The probability that exactly one of them occurs is $11/25$ and the probability of none of them occurring is $2/25$. If $P(T)$ denotes the probability of occurrence of the event T , then

- (1) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$
 (2) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$
 (3) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$
 (4) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$ (IIT-JEE 2011)

2. A ship is fitted with three engines E_1, E_2 , and E_3 . The engines function independently of each other with respective probabilities $1/2, 1/4$, and $1/4$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 , and X_3 denote, respectively, the events that the engines E_1, E_2 , and E_3 are functioning. Which of the following is(are) true?

- (1) $P(X_1^C | X) = \frac{3}{16}$
 (2) $P(\text{exactly two engines of the ship are functioning} | X) = \frac{7}{8}$

$$(3) P(X|X_2) = \frac{5}{16}$$

$$(4) P(X|X_1) = \frac{7}{16}$$

(IIT-JEE 2012)

3. Let X and Y be two events such that $P(X) = \frac{1}{3}$,

$$P(X|Y) = \frac{1}{2} \text{ and } P(Y|X) = \frac{2}{5}. \text{ Then}$$

$$(1) P(Y) = \frac{4}{15} \quad (2) P(X'|Y) = \frac{1}{2}$$

$$(3) P(X \cup Y) = \frac{2}{5} \quad (4) P(X \cap Y) = \frac{1}{5}$$

(JEE Advanced 2017)

Linked Comprehension Type

For Problems 1–3

A fair die is tossed repeatedly until a 6 is obtained. Let X denote the number of tosses required. (IIT-JEE 2009)

1. The probability that $X = 3$ equals

- (1) $25/216$ (2) $25/36$
 (3) $5/36$ (4) $125/216$

2. The probability that $X \geq 3$ equals

- (1) $125/216$ (2) $25/36$
 (3) $5/36$ (4) $25/216$

3. The conditional probability that $X \geq 6$ given $X > 3$ equals

- (1) $125/216$ (2) $25/36$
 (3) $5/36$ (4) $25/216$

For Problems 4 and 5

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears, then 1 ball is drawn at random from U_1 and put into U_2 . However, if tail appears, then 2 balls are drawn at random from U_1 and put into U_2 . Now 1 ball is drawn at random from U_2 .

(IIT-JEE 2011)

4. The probability of the drawn ball from U_2 being white is

- (1) $\frac{13}{30}$ (2) $\frac{23}{30}$ (3) $\frac{19}{30}$ (4) $\frac{11}{30}$

5. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

- (1) $\frac{17}{23}$ (2) $\frac{11}{23}$ (3) $\frac{15}{23}$ (4) $\frac{12}{23}$

For Problems 6 and 7

A box B_1 contains 1 white ball, 3 red balls, and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls, and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls, and 5 black balls. (JEE Advanced 2013)

6. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3 , the probability that all 3 drawn balls are of the same color is

- (1) $82/648$ (2) $90/648$
 (3) $558/648$ (4) $566/648$

7. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

- (1) $116/182$ (2) $126/181$
(3) $65/181$ (4) $55/181$

For Problems 8 and 9

Let n_1 and n_2 be the numbers of red and black balls, respectively, in box I. Let n_3 and n_4 be the numbers of red and black balls, respectively, in box II. **(JEE Advanced 2015)**

8. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $1/3$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is (are)

- (1) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$
(2) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(3) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$
(4) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

9. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $1/3$, then the correct option(s) with the possible values of n_1 and n_2 is (are)

- (1) $n_1 = 4$ and $n_2 = 6$ (2) $n_1 = 2$ and $n_2 = 3$
(3) $n_1 = 10$ and $n_2 = 20$ (4) $n_1 = 3$ and $n_2 = 6$

For Problems 10 and 11

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets

3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 , respectively, after two games. **(JEE Advanced 2016)**

10. $P(X > Y)$ is

- (1) $\frac{1}{4}$ (2) $\frac{5}{12}$
(3) $\frac{1}{2}$ (4) $\frac{7}{12}$

11. $P(X = Y)$ is

- (1) $\frac{11}{36}$ (2) $\frac{1}{3}$
(3) $\frac{13}{36}$ (4) $\frac{1}{2}$

Numerical Value Type

1. Of the three independent events E_1, E_2 , and E_3 , the probability that only E_1 occurs is α , only E_2 occurs is β , and only E_3 occurs is γ . Let the probability p that none of events E_1, E_2 , or E_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0, 1)$. Then

$$\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} = \underline{\hspace{2cm}}.$$

(JEE Advanced 2013)

2. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is _____. **(JEE Advanced 2015)**

3. Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$. Let the events E_1 and E_2 be given by

$$E_1 = \{A \in S : \det. A = 0\} \text{ and}$$

$$E_2 = \{A \in S : \text{Sum of entries of } A \text{ is } 7\}.$$

If a matrix is chosen at random from S , then the conditional probability $P(E_1|E_2)$ equals _____.

(JEE Advanced 2019)

4. Let $|X|$ denote the number of elements in a set X . Let $S = \{1, 2, 3, 4, 5, 6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S , then the number of ordered pairs (A, B) such that $1 \leq |B| < |A|$ equals _____. **(JEE Advanced 2019)**

5. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is NOT less than 0.95, is _____. **(JEE Advanced 2020)**

6. Two fair dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of $14p$ is _____. **(JEE Advanced 2020)**

Answers Key

EXERCISES

Single Correct Answer Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (1) | 2. (2) | 3. (1) | 4. (1) | 5. (2) |
| 6. (2) | 7. (2) | 8. (2) | 9. (3) | 10. (4) |
| 11. (2) | 12. (4) | 13. (3) | 14. (2) | 15. (4) |
| 16. (2) | 17. (1) | 18. (1) | 19. (1) | 20. (2) |
| 21. (3) | 22. (4) | 23. (3) | 24. (1) | 25. (3) |
| 26. (1) | 27. (3) | 28. (3) | 29. (2) | 30. (2) |
| 31. (4) | 32. (3) | 33. (2) | 34. (2) | 35. (1) |
| 36. (1) | 37. (3) | 38. (1) | 39. (1) | 40. (4) |
| 41. (4) | 42. (3) | 43. (2) | 44. (4) | 45. (3) |
| 46. (3) | 47. (3) | 48. (4) | 49. (3) | 50. (2) |
| 51. (3) | 52. (2) | 53. (4) | 54. (4) | 55. (2) |
| 56. (3) | 57. (3) | 58. (4) | 59. (2) | 60. (1) |
| 61. (3) | 62. (1) | 63. (2) | 64. (4) | 65. (3) |
| 66. (1) | 67. (1) | 68. (2) | | |

Multiple Correct Answers Type

- | | | |
|-------------------|-----------------------|-----------------------|
| 1. (1), (3), (4) | 2. (1), (2), (3) | 3. (1), (2), (3) |
| 4. (2), (3) | 5. (3), (4) | 6. (1), (2) |
| 7. (2), (3), (4) | 8. (1), (2), (3), (4) | 9. (1), (2), (3), (4) |
| 10. (1), (2), (4) | 11. (1), (2), (3) | 12. (1), (2), (3) |
| 13. (2), (3) | 14. (1), (2), (3) | 15. (1), (4) |
| 16. (1), (3) | 17. (1), (2), (3) | |

Linked Comprehension Type

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (2) | 2. (3) | 3. (1) | 4. (4) | 5. (4) |
| 6. (2) | 7. (3) | 8. (2) | 9. (4) | 10. (2) |

- | | | | | |
|---------|---------|---------|---------|---------|
| 11. (1) | 12. (3) | 13. (4) | 14. (2) | 15. (1) |
| 16. (1) | 17. (3) | 18. (4) | 19. (1) | 20. (3) |
| 21. (3) | | | | |

Matrix Match Type

- $a \rightarrow p; b \rightarrow r; c \rightarrow q; d \rightarrow p.$
- $a \rightarrow q, r, s; b \rightarrow r; c \rightarrow p, q; d \rightarrow p, q, r, s.$
- $a \rightarrow r, s; b \rightarrow p, q, r, s; c \rightarrow p, q, r, s; d \rightarrow p.$
- $a \rightarrow q, r; b \rightarrow r; c \rightarrow p; d \rightarrow p, q, r, s.$
- (2)
- (4)

Numerical Value Type

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (7) | 2. (1) | 3. (2) | 4. (3) | 5. (6) |
| 6. (5) | 7. (4) | 8. (2) | 9. (3) | 10. (6) |

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Single Correct Answer Type

- | | | | | |
|--------|--------|--------|--------|--------|
| 1. (3) | 2. (1) | 3. (3) | 4. (2) | 5. (1) |
|--------|--------|--------|--------|--------|

Multiple Correct Answers Type

- | | | |
|-------------|-------------|-------------|
| 1. (1), (4) | 2. (2), (4) | 3. (1), (2) |
|-------------|-------------|-------------|

Linked Comprehension Type

- | | | | | |
|---------|---------|-------------|-------------|--------|
| 1. (1) | 2. (2) | 3. (4) | 4. (2) | 5. (4) |
| 6. (1) | 7. (4) | 8. (1), (2) | 9. (3), (4) | |
| 10. (2) | 11. (3) | | | |

Numerical Value Type

- | | | | |
|--------|--------|-----------|-------------|
| 1. (6) | 2. (8) | 3. (0.50) | 4. (422.00) |
| 5. (6) | 6. (8) | | |

Chapter 14

Concept Application Exercises

Exercise 14.1

1. Let E be the event in which all three coins show tail and F be the event in which a coin shows tail. Therefore, $F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$ and $E = \{TTT\}$. Hence, the required probability is

$$\therefore P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{7}$$

$$\begin{aligned} 2. P[B/(A \cup B')] &= \frac{P(B \cap (A \cup B'))}{P(A \cup B')} \\ &= \frac{P(A \cap B)}{P(A) + P(B') - P(A \cap B')} \\ &= \frac{P(A) - P(A \cap B')}{P(A) + P(B') - P(A \cap B')} \\ &= \frac{0.7 - 0.5}{0.8} = \frac{1}{4} \end{aligned}$$

3. Since 4 has appeared on the first, so we are required 4 or 5 or 6 on second dice; hence, required probability is $3/6 = 1/2$.

$$4. P(E) = \frac{4}{8} = \frac{1}{2} \quad [\because \text{favorable cases are } THH, HTH, HHT, HHH]$$

$$P(F) = \frac{4}{8} = \frac{1}{2} \quad [\because \text{favorable cases are } HTH, HHT, HTT, HHH]$$

$$P(E \cap F) = \frac{3}{8}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{1/2} = \frac{3}{4}$$

5. Let M be the event that man is chosen and E be the event that chosen one is employed. From the concept of reduced sample space, we immediately get

$$P(M/E) = \frac{460}{600} = \frac{23}{30}$$

$$\text{Also, } P(E) = \frac{600}{900} = \frac{2}{3}$$

$$P(E \cap M) = \frac{460}{900} = \frac{23}{45}$$

$$\Rightarrow P(E/M) = \frac{23/45}{2/3} = \frac{23}{30}$$

Exercise 14.2

1. Required probability

$$\begin{aligned} &= P(WBWB) + P(BWBW) \\ &= P(W)P(B/W)P(W/WB)P(B/WBW) + \\ &\quad P(B)P(W/B)P(B/BW)P(W/BWB) \\ &= \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{4}{5} \\ &= \frac{1}{14} + \frac{1}{14} = \frac{1}{7} \end{aligned}$$

2. We must have one ace in $(n-1)$ attempts and one ace in the n th attempt.

Let event A be one ace card and $(n-2)$ other than ace cards are drawn in first $(n-1)$ draws.

$$\therefore P(A) = \frac{{}^4C_1 \times {}^{48}C_{n-2}}{{}^{52}C_{n-1}}$$

Let event B be n th card is ace.

Therefore, required probability is

$$P(A \cap B) = P(A)P(B/A)$$

$$\text{Where, } P(B/A) = \frac{{}^3C_1}{{}^{[52-(n-1)]}C_1} = \frac{3}{53-n}$$

$$\begin{aligned} \therefore P(A \cap B) &= \frac{4 \times 48!}{(n-2)!(50-n)!} \times \frac{(n-1)!(53-n)!}{52!} \times \frac{3}{53-n} \\ &= \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13} \end{aligned}$$

3. Since the question is multiple choice in which one or more than one options are correct, correct options may be one, two, three or four.

$\therefore$ Total number of ways of ticking one or more alternatives out of four $= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 15$

Out of these 15 combinations, only one combination is correct.

$\therefore$ Probability of ticking the alternative correctly at the first trial,

$$P(T_1) = \frac{1}{15}$$

Probability of ticking the answer correctly in second trial,

$$\begin{aligned} P(T_1' \cap T_2) &= P(T_1')P(T_2/T_1') \\ &= \frac{14}{15} \times \frac{1}{14} = \frac{1}{15} \end{aligned}$$

Probability of ticking the answer correctly in third trial,

$$\begin{aligned} P(T_1' \cap T_2' \cap T_3) &= P(T_1')P(T_2'/T_1')P(T_3/T_1'T_2') \\ &= \frac{14}{15} \times \frac{13}{14} \times \frac{1}{13} = \frac{1}{15} \end{aligned}$$

Thus, the probability that the candidate will get marks on the question if he is allowed up to three trials

$$= \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{1}{5}$$

Exercise 14.3

$$1. P(A) = \frac{3}{8} \text{ and } P(B) = \frac{1}{2}$$

$$P(A)P(B) = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$$

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4} \neq P(A)P(B)$$

Hence, A and B are not independent.

$$2. \text{ It is given that } P(A) = \frac{1}{2}, P(B) = \frac{7}{12},$$

$$\text{and } P(\text{not } A \text{ or not } B) = \frac{1}{4}$$

$$\begin{aligned} \Rightarrow P(A' \cup B') &= \frac{1}{4} \\ \Rightarrow P((A \cap B)') &= \frac{1}{4} & [\because A' \cup B' = (A \cap B)'] \\ \Rightarrow 1 - P(A \cap B) &= \frac{1}{4} \\ \text{or } P(A \cap B) &= \frac{3}{4} \end{aligned} \quad (1)$$

However,

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \quad (2)$$

$$\text{Here, } \frac{3}{4} \neq \frac{7}{24}$$

$$\therefore P(A \cap B) \neq P(A) \cdot P(B)$$

Therefore, A and B are not independent events.

3. Probability of first card to be a king, $P(A) = 4/52$ and probability of second to be a king, $P(B) = 3/51$. Hence, required probability is

$$P(A \cap B) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

4. Here $P(A) = 0.4$ and $P(\bar{A}) = 0.6$. Probability that A does not happen at all is $(0.6)^3$. Thus, required probability is $1 - (0.6)^3 = 0.784$.

5. P (six balls drawn follow the pattern $WBWBWB$)

$$= \left(\frac{3}{6}\right)^6 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$\therefore P(\text{six balls drawn follow the pattern } BWBWBW)$$

$$= \left(\frac{3}{6}\right)^6 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$\therefore \text{Required probability} = \frac{1}{64} + \frac{1}{64} = \frac{1}{32}$$

6. Let probability of success in a trial is ' p '

$$\begin{aligned} \text{Then } P(X=4) &= {}^{10}C_4 p^4 (1-p)^6 \\ P(X) &= {}^{10}C_4 [4p^3(1-p)^6 - 6(1-p)^5 p^4] \\ &= {}^{10}C_4 p^3 (1-p)^5 (4-10p) \end{aligned}$$

For maximum $P'(X) = 0$

$$\Rightarrow P(X) = \frac{2}{5}$$

$$\Rightarrow P'(X) = \frac{3}{5}$$

7. Let E_1 be the events that man will be selected and E_2 the events that woman will be selected. Then,

$$P(E_1) = \frac{1}{4}, \text{ so } P(\bar{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(E_2) = \frac{1}{3}, \text{ so } P(\bar{E}_2) = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly E_1 and E_2 are independent events. So,

$$P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \times P(\bar{E}_2) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$

$$8. P(K) = \frac{7}{15} \therefore P(\bar{K}) = 1 - \frac{7}{15} = \frac{8}{15}$$

$$P(H) = \frac{7}{10} \therefore P(\bar{H}) = 1 - \frac{7}{10} = \frac{3}{10}$$

Hence, required probability is

$$P(\bar{K} \cap \bar{H}) = P(\bar{K}) P(\bar{H}) = \frac{8}{15} \times \frac{3}{10} = \frac{24}{150} = \frac{4}{25}$$

9. Probability for an incorrect digit is p . Hence, probability for a correct digit is $1 - p$. Hence, probability for 8 correct digit is $(1 - p)^8$. Hence, required probability is $1 - (1 - p)^8$.

10. Required probability is

$$\begin{aligned} &P(\bar{A}_1 \cap A_2 \cap A_3) + P(A_1 \cap \bar{A}_2 \cap A_3) \\ &= P(\bar{A}_1)P(A_2)P(A_3) + P(A_1)P(\bar{A}_2)P(A_3) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4} \end{aligned}$$

11. Let p be the chance of cutting a spade and q be the chance of not cutting a spade from the pack of 52 cards.

Then $p = 13/52 = 1/4$ and $q = 1 - 1/4 = 3/4$.

Now, A will win a prize if the cuts spade at 1st, 4th, 7th, 10th turns etc. Note that A will get a second chance if A, B and C all fail to cut spade once and then A cuts a spade at the 4th turn.

Similarly, he will cut a spade at the 7th turn when each of A, B, C fail to cut spade twice etc. Hence, A 's chance of winning the prize

$$\begin{aligned} &= p + q^3 p + q^6 p + q^9 p + \dots \\ &= \frac{p}{1 - q^3} = \frac{\frac{1}{4}}{1 - \left(\frac{3}{4}\right)^3} = \frac{16}{37} \end{aligned}$$

Similarly, B 's chance

$$\begin{aligned} &= (qp + q^4 p + q^7 p + \dots) \\ &= q(p + q^3 p + q^6 p) = 12/37 \end{aligned}$$

and C 's chance $= 1 - (16/37) - (12/37) = 9/37$

12. Let

W : event of drawing a white ball at any draw; and

B : event of drawing a black ball at any draw

Then

$$P(W) = \frac{a}{a+b} \text{ and } P(B) = \frac{b}{a+b}$$

$P(A \text{ wins the game}) = P(W \text{ or } BBW \text{ or } BBBBW \text{ or } \dots)$

$$\begin{aligned} &= P(W) + P(BBW) + P(BBBBW) + \dots \\ &= P(W) + P(B)P(B)P(W) + P(B)P(B)P(B)P(B)P(W) + \dots \\ &= P(W) + P(W)(P(B))^2 + P(W)(P(B))^4 + \dots \\ &= \frac{P(W)}{1 - (P(B))^2} = \frac{a(a+b)}{a^2 + 2ab} = \frac{a+b}{a+2b} \end{aligned}$$

$$\text{Also, } P(B \text{ wins the game}) = 1 - \frac{a+b}{a+2b} = \frac{a}{a+2b}$$

According to the given condition,

$$\Rightarrow \frac{a+b}{a+2b} = 3 \times \frac{b}{a+2b}$$

$$\Rightarrow a = 2b$$

$$\Rightarrow a : b = 2 : 1$$

Exercise 14.4

1. The urn contains 5 red and 5 black balls.

Let a red ball be drawn in the first attempt. Therefore,

$$P(\text{drawing a red ball}) = \frac{5}{10} = \frac{1}{2}$$

If two red balls are added to the urn, then the urn contains 7 red and 5 black balls. Thus,

$$P(\text{drawing a red ball}) = \frac{7}{12}$$

Let a black ball be drawn in the first attempt. Then

$$P(\text{drawing a black ball in the first attempt}) = \frac{5}{10} = \frac{1}{2}$$

If two black balls are added to the urn, then the urn contains 5 red and 7 black balls.

$$P(\text{drawing a red ball}) = \frac{5}{12}$$

Therefore, probability of drawing the second ball as red is

$$\frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{1}{2} \left(\frac{7}{12} + \frac{5}{12} \right) = \frac{1}{2} \times 1 = \frac{1}{2}$$

2. If third ball is red, then in first two draws, there will be either no red ball or one red ball.

Let events

R_0 = in first two draws, no red ball is drawn

R_1 = in first two draws, one red ball is drawn.

R = in third draw, red ball is drawn.

So, from total probability theorem,

$$\begin{aligned} P(R) &= P(R_0) P(R/R_0) + P(R_1) P(R/R_1) \\ &= \frac{{}^6C_2}{{}^8C_2} \cdot \frac{{}^2C_1}{{}^6C_1} + \frac{{}^6C_1 {}^1C_1}{{}^8C_2} \cdot \frac{{}^1C_1}{{}^6C_1} \\ &= \frac{15}{28} \cdot \frac{2}{6} + \frac{6 \times 2}{28} \cdot \frac{1}{6} \\ &= \frac{1}{4} \end{aligned}$$

3. We have $P(G) = 1/3$ and $P(B) = 2/3$

$$P(A/G) = 0.25 \text{ and } P(A/B) = 0.28$$

Here, A is the event that student chosen will get a first class.

So, using total probability theorem,

$$\begin{aligned} P(A) &= P(B) P(A/B) + P(G) P(A/G) \\ &= \frac{2}{3} \times 0.28 + \frac{1}{3} \times 0.25 \\ &= 0.27 \end{aligned}$$

4. Let C be the event of the selected number being composite and E be the event of there being no remainder.

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{25} = \frac{3}{5}, P(\bar{C}) = \frac{2}{5}$$

$$P(E/C) = \frac{4}{15}, P(E/\bar{C}) = \frac{1}{10}$$

$$\begin{aligned} P(E) &= P(C) P(E/C) + P(\bar{C}) P(E/\bar{C}) \\ &= \frac{3}{5} \times \frac{4}{15} + \frac{2}{5} \times \frac{1}{10} = \frac{4}{25} + \frac{1}{25} = \frac{1}{5} = 0.2 \end{aligned}$$

5. Let A_1 and A_2 be the events that the specific home is left unlocked, and is left locked, respectively. Then $P(A_1) = 0.4$, $P(A_2) = 0.6$.

Let A be the event that the real estate man get into the specific home. Then

$$P(A/A_1) = 1, P(A/A_2) = \frac{{}^7C_2}{{}^8C_3} = \frac{3}{8}$$

$$\begin{aligned} \Rightarrow P(A) &= P(A_1) P(A/A_1) + P(A_2) P(A/A_2) \\ &= (0.4)(1) + (0.6)\left(\frac{3}{8}\right) = \frac{4}{10} + \frac{18}{80} = \frac{5}{8} \end{aligned}$$

6. Let W_1 (B_1) be the event that a white (a black) ball is drawn in the first draw and let W be the event that white ball is drawn in the second draw. Then

$$\begin{aligned} P(W) &= P(B_1) P(W/B_1) + P(W_1) P(W/W_1) \\ &= \frac{n}{m+n} \frac{m}{m+n+k} + \frac{m}{m+n} \frac{m+k}{m+n+k} \\ &= \frac{m(n+m+k)}{(m+n)(m+n+k)} \\ &= \frac{m}{m+n} \end{aligned}$$

7. Let us define the following events

A : 4 white balls are drawn in first six draws

B : 5 white balls are drawn in first six draws

C : 6 white balls are drawn in first six draws

E : exactly one white ball is drawn in next two draws (i.e., one white and one red)

Then

$$P(E) = P(E/A) P(A) + P(E/B) P(B) + P(E/C) P(C)$$

But

$$P(E/C) = 0 \quad [\text{as there are only 6 white balls in the bag}]$$

$$\begin{aligned} \therefore P(E) &= P(E/A) P(A) + P(E/B) P(B) \\ &= \frac{{}^{10}C_1 \times {}^2C_1}{{}^{12}C_2} \frac{{}^{12}C_2 \times {}^6C_4}{{}^{18}C_6} + \frac{{}^{11}C_1 \times {}^1C_1}{{}^{12}C_1} \frac{{}^{12}C_1 \times {}^6C_5}{{}^{18}C_6} \end{aligned}$$

Exercise 14.5

1. Let A_1 be the event that missing card is spade and A_2 be event that missing card is non-spade. Then,

$$P(A_1) = \frac{1}{4}, P(A_2) = \frac{3}{4}$$

Let A be the event that 2 spade cards are drawn from the remaining cards. Then,

$$P\left(\frac{A}{A_1}\right) = \frac{{}^{12}C_2}{{}^{51}C_2} \text{ and } P\left(\frac{A}{A_2}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$\begin{aligned} \Rightarrow P(A) &= P(A_1) P\left(\frac{A}{A_1}\right) + P(A_2) P\left(\frac{A}{A_2}\right) \\ &= \frac{1}{4} \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{3}{4} \frac{{}^{13}C_2}{{}^{51}C_2} \\ &= \frac{1}{4 \times {}^{51}C_2} [{}^{12}C_2 + 3 \times {}^{13}C_2] \end{aligned}$$

Now,

$$\begin{aligned} P\left(\frac{A_1}{A}\right) &= \frac{P(A_1) P\left(\frac{A}{A_1}\right)}{P(A)} \\ &= \frac{\frac{1}{4} \frac{{}^{12}C_2}{{}^{51}C_2}}{\frac{1}{4 \times {}^{51}C_2} [{}^{12}C_2 + 3 \times {}^{13}C_2]} \\ &= \frac{11}{50} \end{aligned}$$

2. Let E_1 , E_2 , and E_3 be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin. Then

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads. Therefore,

$$P(A/E_1) = P(\text{coin showing heads, given that it is a two-headed coin}) = 1$$

Probability of heads coming up, given that it is a biased coin = 75%

$$\begin{aligned} \therefore P(A/E_1) &= P(\text{coin showing heads, given that it is a biased coin}) \\ &= \frac{75}{100} = \frac{3}{4} \end{aligned}$$

Since the third coin is unbiased, the probability that it shows heads is always $1/2$.

$$\begin{aligned} \therefore P(A/E_3) &= P(\text{coin showing heads, given that it is an unbiased coin}) \\ &= \frac{1}{2} \end{aligned}$$

The probability that the coin is two-headed, given that it shows heads, is given by

$$P(E_1/A)$$

By using Bayes' theorem, we obtain

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)} \\ &= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{4}{9} \end{aligned}$$

3. Let E_1 and E_2 be the events such that

E_1 : A speaks truth

E_2 : A speaks false

Let X be the event that a head appears.

$$P(E_1) = \frac{4}{5}$$

$$\therefore P(E_2) = 1 - P(E_1) = 1 - \frac{4}{5} = \frac{1}{5}$$

If a coin is tossed, then it may result in either head (H) or tail (T).

The probability of getting a head is $\frac{1}{2}$ whether A speaks truth or not. Therefore,

$$P(X/E_1) = P(X/E_2) = \frac{1}{2}$$

The probability that there is actually a head is given by $P(E_1/X)$.

$$\begin{aligned} P(E_1/X) &= \frac{P(E_1) \cdot P(X/E_1)}{P(E_1) \cdot P(X/E_1) + P(E_2) \cdot P(X/E_2)} \\ &= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}} = \frac{4}{5} \end{aligned}$$

4. Let A_i ($i = 1, 2, 3, 4$) be the event that the urn contains 2, 3, 4, or 5 white balls and E the event that two white balls are drawn. Since the four events A_1, A_2, A_3, A_4 are equally likely, therefore $P(A_i) = 1/4$, $i = 1, 2, 3, 4$. Now, the probability that the urn contains 2 white balls and both have been drawn is

$$P(E/A_1) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$$

Similarly,

$$P(E/A_2) = \frac{{}^3C_2}{{}^5C_2} = \frac{3}{10},$$

$$P(E/A_3) = \frac{{}^4C_2}{{}^5C_2} = \frac{3}{5}$$

$$P(E/A_4) = \frac{{}^5C_2}{{}^5C_2} = 1$$

Hence, the required probability is

$$\begin{aligned} P(A_4/E) &= \frac{P(A_4)P(E/A_4)}{\sum_{i=1}^4 P(A_i)P(E/A_i)} \\ &= \frac{\frac{1}{4} \times 1}{\frac{1}{4} \left(\frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1 \right)} = \frac{1}{2} \end{aligned}$$

5. Let E_1, E_2 , and E_3 denote the events of selecting boxes A, B, C , respectively, and A be the event that a screw selected at random is defective. Then,

$$P(E_1) = P(E_2) = P(E_3) = 1/3$$

$$\therefore P(A/E_1) = \frac{1}{5}, P(A/E_2) = \frac{1}{6}, P(A/E_3) = \frac{1}{7}$$

By Bayes's rule, the required probability is

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} \\ &= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{7}} = \frac{42}{107} \end{aligned}$$

6. Let A, E_1 , and E_2 , respectively, denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription. Therefore,

$$P(A) = 0.40$$

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A/E_1) = 0.40 \times 0.70 = 0.28$$

$$P(A/E_2) = 0.40 \times 0.75 = 0.30.$$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by $P(E_1/A)$.

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} \\ &= \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30} = \frac{14}{29} \end{aligned}$$

7. Let events

A : Event that first man speaks truth

B : Event that second man speaks truth

R : Day is rainy

$$\therefore P(A) = \frac{4}{5}, P(B) = \frac{2}{3}, P(R) = \frac{3}{4}$$

$\therefore$ Required probability

$$\begin{aligned} &= \frac{P(A \cap B) \cdot P(R)}{P(A \cap B) \cdot P(R) + P(A' \cap B') \cdot P(R')} \\ &= \frac{\frac{4}{5} \times \frac{2}{3} \times \frac{3}{4}}{\frac{4}{5} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{5} \times \frac{1}{3} \times \frac{1}{4}} = \frac{24}{25} \end{aligned}$$

Exercise 14.6

1. According to question,

$${}^nC_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{n-6} = {}^nC_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{n-8}$$

$$\text{or } {}^nC_6 \left(\frac{1}{2}\right)^n = {}^nC_8 \left(\frac{1}{2}\right)^n$$

$$\text{or } {}^nC_6 = {}^nC_8 = {}^nC_{n-8}$$

$$\Rightarrow 6 = n - 8 \text{ or } n = 14$$

2. Given that $p = 1/6$, $q = 5/6$. Therefore, required probability is

$$\begin{aligned} P(0) + P(1) + P(2) &= {}^4C_0 \left(\frac{5}{6}\right)^4 + {}^4C_1 \left(\frac{5}{6}\right)^3 \frac{1}{6} + {}^4C_2 \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2 \\ &= \frac{425}{432} \\ &= 0.984 \end{aligned}$$

3. Let p be the probability that a student selected at random is a swimmer, which is given by $1 - 1/5 = 4/5$. The number of students selected is 5. Probability that exactly 4 students are swimmers is

$${}^5C_4 p^4 q = 5 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right) = \left(\frac{4}{5}\right)^4$$

4. The probability p_1 of winning the best of three games is equal to the sum of the probability of winning two games and the probability of winning three games, which is given by

$$\begin{aligned} p_1 &= {}^3C_2 (0.6) (0.4)^2 + {}^3C_3 (0.4)^3 \\ &= 0.288 + 0.064 = 0.352 \quad [\text{Using binomial distribution}] \end{aligned}$$

Similarly, the probability of winning the best five games is

$$\begin{aligned} p_2 &= \text{Probability of winning three games} \\ &\quad + \text{Probability of winning four games} \\ &\quad + \text{Probability of winning five games} \\ &= {}^5C_3 (0.6)^2 (0.4)^3 + {}^5C_4 (0.6) (0.4)^4 + {}^5C_5 (0.4)^5 \\ &= 0.2304 + 0.0768 + 0.01024 \\ &= 0.31744 \end{aligned}$$

As $p_1 > p_2$, A must choose the first offer, i.e., best of three games.

5. The given numbers are 00, 01, 02, ..., 99. There are total 100 numbers, out of which the numbers, the product of whose digits is 18, are 29, 36, 63, and 92.

$$\therefore p = P(E) = \frac{4}{100} = \frac{1}{25}$$

$$\Rightarrow q = 1 - p = \frac{24}{25}$$

From binomial distribution,

$P(E \text{ occurring at least 3 times})$

$$\begin{aligned} &= P(E \text{ occurring 3 times}) + P(E \text{ occurring 4 times}) \\ &= {}^4C_3 p^3 q + {}^4C_4 p^4 \\ &= 4 \times \left(\frac{1}{25}\right)^3 \left(\frac{24}{25}\right) + \left(\frac{1}{25}\right)^4 \\ &= \frac{97}{(25)^4} \end{aligned}$$

Exercises

Single Correct Answer Type

$$1. (1) P(B_1) = \frac{{}^6C_1}{{}^{10}C_1} = \frac{6}{10} = \frac{3}{5}$$

$$P(B_2 / B_1) = \frac{5}{9} \quad (B_2 = \text{black})$$

$$\therefore P(B_1 \cap B_2) = P(B_1) P(B_2 / B_1) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$$

2. (2) Consider the following events:

A : getting a card with mark I in first draw

B : getting a card with mark I in second draw

C : getting a card with mark T in this draw

Then, the required probability is

$$P(A \cap B \cap C) = P(A) P(B/A) P(C/A \cap B)$$

$$= \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18} = \frac{5}{38}$$

3. (1) Let the number selected be xy . Then

$$x + y = 9, 0 \leq x, y \leq 9$$

$$\text{and } xy = 0 \Rightarrow x = 0, y = 9$$

$$\text{or } y = 0, x = 9$$

$$P(x_1 = 9 / x_2 = 0) = \frac{P(x_1 = 9 \cap x_2 = 0)}{P(x_2 = 0)}$$

$$\text{Now, } P(x_2 = 0) = \frac{19}{100}$$

$$\text{and } P(x_1 = 9 \cap x_2 = 0) = \frac{2}{100}$$

$$\Rightarrow P(x_1 = 9 / x_2 = 0) = \frac{2/100}{19/100} = \frac{2}{19}$$

4. (1) $P(A \cap B') = P(A) - P(A \cap B) = 0.20$

Also,

$$P(A' \cap B) = P(B) - P(A \cap B) = 0.15$$

$$\Rightarrow P(A) + P(B) - 2P(A \cap B) = 0.35$$

Now,

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$\Rightarrow 0.1 = 1 - P(A) - P(B) + P(A \cap B)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = 0.9$$

$$\Rightarrow P(A \cap B) = 0.9 - 0.35 = 0.55$$

$$\text{and } P(A) = 0.75, P(B) = 0.70$$

Now,

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.55}{0.70}$$

5. (2) Consider the following events:

A : Father has at least one boy

B : Father has 2 boys and one girl

Then,

A = one boy and 2 girls, 2 boys and one girl, 3 boys and no girl

$A \cap B$ = 2 boys and one girl

Now, the required probability is

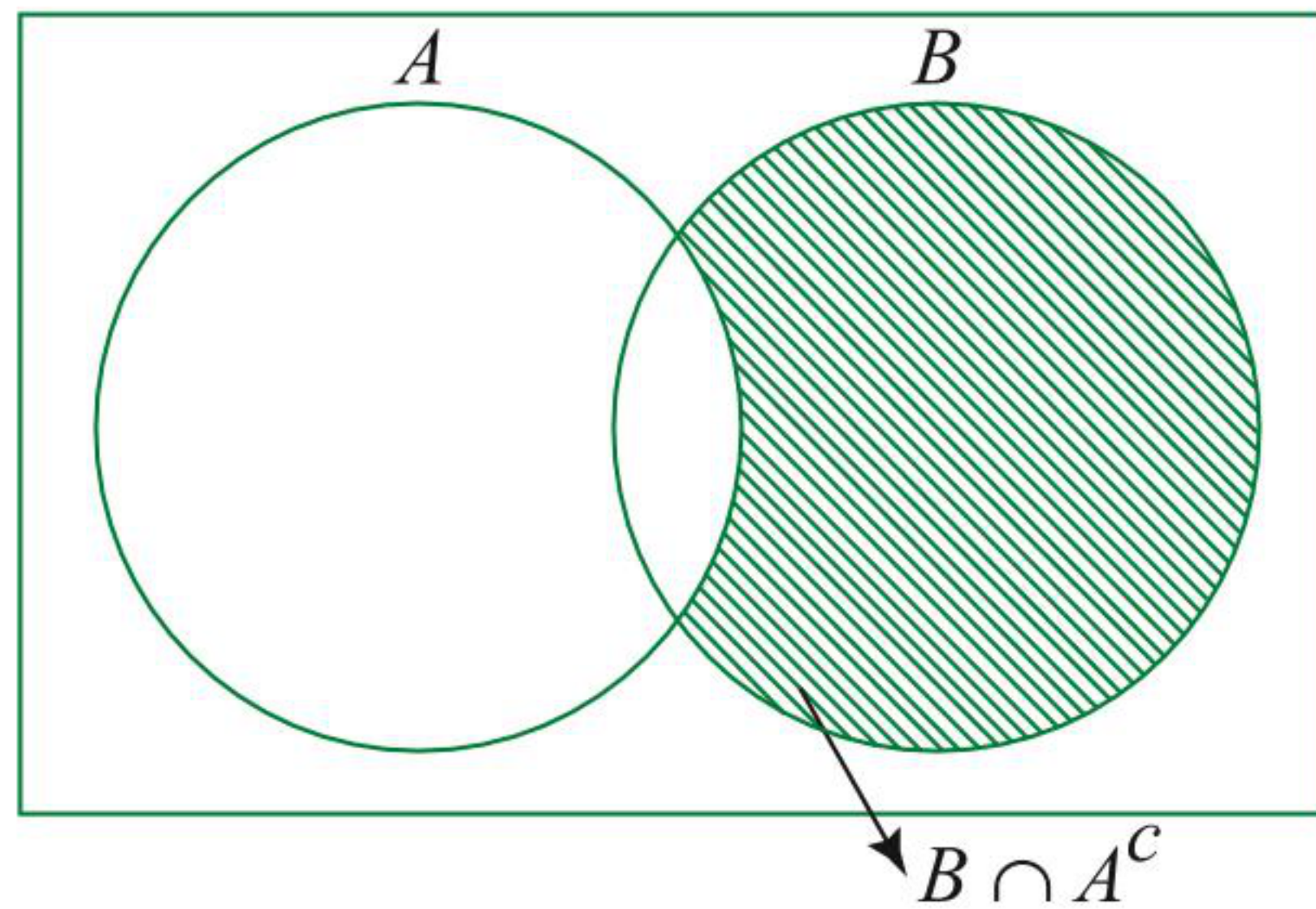
$$= P(A/B) = \frac{P(A \cap B)}{P(A)} = \frac{1}{3}$$

6. (2) We have,

$$P(A) = \frac{40}{100}, P(B) = \frac{25}{100} \text{ and } P(A \cap B) = \frac{15}{100}$$

$$\text{So, } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{15/100}{40/100} = \frac{3}{8}$$

7. (2)



$$P(A) = \frac{1}{4}, P(A \cup B) = \frac{1}{2}$$

$$\begin{aligned} P\left(\frac{B}{A^c}\right) &= \frac{P(B \cap A^c)}{P(A^c)} \\ &= \frac{P(A \cup B) - P(A)}{1 - P(A)} \\ &= \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \end{aligned}$$

[$\because P(A \cup B) + P(B) - P(A \cap B)$]

8. (2) $P(S \cap F) = 0.0006$, where S is the event that the motor cycle is stolen and F is the event that the motor cycle is found. Therefore,

$$P(S) = 0.0015$$

$$P(F/S) = \frac{P(F \cap S)}{P(S)} = \frac{6 \times 10^{-4}}{15 \times 10^{-4}} = \frac{2}{5}$$

9. (3) Let A be the event that 11 is picked and B be the event that sum is even. The number of ways of selecting 11 along with one more odd number is $n(A \cap B) = {}^7C_1$.

The number of ways of selecting either two even numbers or selecting two odd numbers is $n(B) = 1 + {}^8C_2$.

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{7}{29} = 0.24$$

10. (4) A : exactly one ace

B : both aces

E : $A \cup B$

$$P(B/A \cup B) = \frac{{}^4C_2}{{}^4C_1 {}^{12}C_1 + {}^4C_2} = \frac{6}{54} = \frac{1}{9}$$

11. (2) The sum of the digits can be 7 in the following ways: 07, 16, 25, 34, 43, 52, 61, 70.

$$\therefore (A = 7) = \{07, 16, 25, 34, 43, 52, 61, 70\}$$

Similarly,

$$(B = 0) = \{00, 01, 02, \dots, 10, 20, 30, \dots, 90\}$$

Thus,

$$(A = 7) \cap (B = 0) = \{09, 70\}$$

$$\therefore P((A = 7) \cap (B = 0)) = \frac{2}{100}, P((B = 0)) = \frac{19}{100}$$

Hence,

$$\begin{aligned} P(A = 7 | B = 0) &= \frac{P((A = 7) \cap (B = 0))}{P(B = 0)} \\ &= \frac{2}{19} = \frac{2}{19} \end{aligned}$$

12. (4) Consider two events as follows:

A_i : getting number i on first die

B_i : getting a number more than i on second die

The required probability is

$$\begin{aligned} &P(A_1 \cap B_1) + P(A_2 \cap B_2) + P(A_3 \cap B_3) + P(A_4 \cap B_4) \\ &+ P(A_5 \cap B_5) = \sum_{i=1}^5 P(A_i \cap B_i) = \sum_{i=1}^5 P(A_i) P(B_i) \\ &[\because A_i, B_i \text{ are independent}] \\ &= \frac{1}{6} [P(B_1) + P(B_2) + \dots + P(B_5)] \\ &= \frac{1}{6} \left(\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6} \right) = \frac{5}{12} \end{aligned}$$

13. (3) 18 draws are required for 2 aces. This means in the first 17 draws, there is one ace and 16 other cards, and 18th draw produces an ace. So, the required probability is

$$\frac{{}^{48}C_{16} \times {}^4C_1}{{}^{52}C_{17}} \times \frac{3}{35} = \frac{561}{15925}$$

14. (2) The required probability is

$$\begin{aligned} &\frac{n^2}{{}^{2n}C_2} \frac{(n-1)^2}{{}^{2n-2}C_2} \frac{(n-2)^2}{{}^{2n-4}C_2} \dots \frac{2^2}{{}^4C_2} \frac{1^2}{{}^2C_2} \\ &= \frac{(1 \times 2 \times 3 \times 4 \times \dots \times (n-1)n)^2}{\frac{(2n)!}{2^n}} = \frac{2^n (n!)^2}{(2n)!} = \frac{2^n}{{}^{2n}C_n} \end{aligned}$$

15. (4) Let the probability for getting an odd number be p . Therefore, the probability for getting an even number is $2p$.

$$\therefore p + 2p = 1 \text{ or } 3p = 1 \text{ or } p = \frac{1}{3}$$

Sum of two numbers is even means either both are odd or both are even. Therefore, the required probability is

$$\frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{1}{9} + \frac{4}{9} = \frac{5}{9}$$

16. (2) If A, B, C represent events that the student is successful in tests I, II, III, respectively, then the probability that the student is successful is

$$\begin{aligned} &P[(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A \cap B \cap C)] \\ &= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A \cap B \cap C) \\ &= P(A) P(B) P(C') + P(A) P(B') P(C) + P(A) P(B) P(C) \\ &[\because A, B, C \text{ are independent events}] \\ &= pq \left(1 - \frac{1}{2} \right) + p(1-q) \frac{1}{2} + pq \frac{1}{2} \\ &= pq + \frac{1}{2} p - \frac{1}{2} pq \\ &= \frac{1}{2} (pq + p) \\ &\therefore \frac{1}{2} p(1+q) = \frac{1}{2} \\ &\text{or } p(1+q) = 1 \end{aligned}$$

$$\begin{aligned}
 17. (1) \quad & P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, \text{ and } P(C) = \frac{1}{4} \\
 \therefore \quad & P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}, P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3} \\
 & P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}
 \end{aligned}$$

Therefore, the required probability is

$$\begin{aligned}
 1 - P(\bar{A})P(\bar{B})P(\bar{C}) &= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \\
 &= 1 - \frac{1}{4} = \frac{3}{4}
 \end{aligned}$$

18. (1) We are given that

$$\begin{aligned}
 P(A \cap B) &= P(A)P(B) \\
 P(B \cap C) &= P(B)P(C) \\
 P(C \cap A) &= P(C)P(A) \\
 P(A \cap B \cap C) &= P(A)P(B)P(C)
 \end{aligned}$$

We have,

$$\begin{aligned}
 P(A \cap (B \cap C)) &= P(A \cap B \cap C) = P(A)P(B)P(C) \\
 &= P(A)P(B \cap C)
 \end{aligned}$$

$\Rightarrow$ A and $B \cap C$ are independent

Therefore, S_2 is true. Also,

$$\begin{aligned}
 P[(A \cap (B \cup C))] &= P[(A \cap B) \cup (A \cap C)] \\
 &= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)] \\
 &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\
 &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\
 &= P(A)[P(B) + P(C) - P(B)P(C)] \\
 &= P(A)[P(B) + P(C) - P(B \cap C)] \\
 &= P(A)P(B \cup C)
 \end{aligned}$$

Therefore, A and $B \cup C$ are independent.

19. (1) The ratios of the ships A , B , and C for arriving safely are 2:5, 3:7, and 6:11, respectively. Therefore, the probability of ship A for arriving safely is $2/(2+5) = 2/7$.

Similarly, for B the probability is $3/(3+7) = 3/10$ and for C the probability is $6/(6+11) = 6/17$.

Therefore, the probability of all the ships for arriving safely is $(2/7) \times (3/10) \times (6/17) = 18/595$.

20. (2) Let event A be drawing 9 cards which are not ace and B be drawing an ace card. Therefore, the required probability is $P(A \cap B) = P(A) \times P(B)$

Now, there are four aces and 48 other cards. Hence,

$$P(A) = \frac{{}^{48}C_9}{{}^{52}C_9}$$

After having drawn 9 non-ace cards, the 10th card must be ace. Hence,

$$P(B) = \frac{{}^4C_1}{{}^{42}C_1} = \frac{4}{42}$$

Hence,

$$P(A \cap B) = \frac{{}^{48}C_9}{52} \times \frac{4}{42}$$

21. (3) Out of 5 horses, only one is the winning horse. The probability that Mr. A selected that losing horse is $4/5 \times 3/4$. Therefore, the required probability is

$$1 - \frac{4}{5} \times \frac{3}{4} = 1 - \frac{3}{5} = \frac{2}{5}$$

22. (4) We have,

$$\begin{aligned}
 P(\overline{A \cup B}) &= \frac{1}{4}, P(A \cap B) = \frac{1}{4} \\
 P(\bar{A}) &= \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4} \\
 \therefore \quad P(\overline{A \cup B}) &= 1 - P(A \cup B) \\
 &= 1 - [P(A) + P(B) - P(A \cap B)] \\
 \Rightarrow \quad \frac{1}{4} &= 1 - \frac{3}{4} - P(B) + \frac{1}{4} \\
 \Rightarrow \quad P(B) &= 1 - \frac{1}{2} - \frac{1}{4} = \frac{6-3-1}{4} = \frac{2}{4} = \frac{1}{2}
 \end{aligned}$$

Since $P(A \cap B) = P(A)P(B)$ and $P(A) \neq P(B)$, therefore A and B are independent but not equally likely.

23. (3) Total number of the students is 80. Total number of girls is 25. Total number of boys is 55. There are 10 rich, 70 poor, 20 intelligent students in the class. Therefore, required probability is

$$\frac{1}{4} \times \frac{1}{8} \times \frac{25}{80} = \frac{5}{512}$$

(I) (R) (G)

24. (1) $P(A \cap C) = P(A)P(C)$

$$\text{or } \frac{1}{20} = \frac{1}{5}P(C)$$

$$\text{or } P(C) = \frac{1}{4}$$

Now,

$$\begin{aligned}
 P(B \cup C) &= \frac{1}{6} + \frac{1}{4} - P(B \cap C) \\
 P(B \cap C) &= \frac{5}{12} - \frac{3}{8} = \frac{1}{24}P(B)P(C)
 \end{aligned}$$

Therefore, B and C are independent.

25. (3) $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$$\Rightarrow 0.7 = 0.4 + p - 0.4p$$

$$\therefore 0.6p = 0.3 \text{ or } p = \frac{1}{2}$$

26. (1) The required probability is

$$= \frac{2}{9} \times \frac{1}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times 1 \times 1 \times 1 = \frac{1}{1260}$$

27. (3) The total number of digits in any number at the unit's place is 10. Therefore,

$$n(S) = 10$$

If the last digit in product is 1, 3, 5 or 7, then it is necessary that the last digit in each number must be 1, 3, 5 or 7. Therefore,

$$n(A) = 4$$

$$\therefore P(A) = \frac{4}{10} = \frac{2}{5}$$

Hence, the required probability is $(2/5)^4 = 16/625$.

28. (3) The probabilities of solving the question by these three students are $1/3$, $2/7$ and $3/8$, respectively. Therefore,

$$P(A) = \frac{1}{3}; P(B) = \frac{2}{7}; P(C) = \frac{3}{8}$$

Then probability of question solved by only one student is

$$\begin{aligned}
 &P((\bar{A}\bar{B}\bar{C} \text{ or } \bar{A}\bar{B}C \text{ or } \bar{A}\bar{B}\bar{C})) \\
 &= P(\bar{A})P(\bar{B})P(\bar{C}) + P(\bar{A})P(\bar{B})P(C) + P(\bar{A})P(B)\bar{P}(\bar{C})
 \end{aligned}$$

$$\begin{aligned}
&= P(A)P(\bar{B})P(\bar{C}) + P(\bar{A})P(B)P(\bar{C}) + P(\bar{A})P(\bar{B})P(C) \\
&= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{2}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{5}{7} \times \frac{3}{8} \\
&= \frac{25 + 20 + 30}{168} = \frac{25}{56}
\end{aligned}$$

29. (2) Probability of getting 2 heads in the first 5 trials is

$${}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = \frac{10}{32} = \frac{5}{16}$$

Therefore, the probability that third head appears on the sixth trial is $5/16 \times 1/2 = 5/32$.

30. (2) Let H denote the head, T the tail and $*$ any of the head or tail. Then, $P(H) = 1/2$, $P(T) = 1/2$ and $P(*) = 1$. For at least four consecutive heads, we should have any of the following patterns:

	Probability
(i) $HHHH****$	$(1/2)^4 \times 1 = 1/16$
(ii) $THHHH**$	$(1/2)^5 = 1/32$
(iii) $*THHHH*$	$(1/2)^5 = 1/32$
(iv) $**THHHH$	$(1/2)^5 = 1/32$

Since all the above cases are mutually exclusive, the probability of getting at least four consecutive heads (on adding) is $1/16 + 3/32 = 5/32$.

31. (4) The probability that the first critic favours the book is

$$P(E_1) = \frac{5}{5+2} = \frac{5}{7}$$

The probability that the second critic favours the book is

$$P(E_2) = \frac{4}{4+3} = \frac{4}{7}$$

The probability that the third critic favours the book is

$$P(E_3) = \frac{3}{3+4} = \frac{3}{7}$$

Majority will be in favor of the book if at least two critics favor the book. Hence, the probability is

$$\begin{aligned}
&P(E_1 \cap E_2 \cap \bar{E}_3) + P(E_1 \cap \bar{E}_2 \cap E_3) \\
&\quad + P(\bar{E}_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) \\
&= P(E_1)P(E_2)P(\bar{E}_3) + P(E_1)P(\bar{E}_2)P(E_3) \\
&\quad + P(\bar{E}_1)P(E_2)P(E_3) + P(E_1)P(E_2)P(E_3) \\
&= \frac{5}{7} \times \frac{4}{7} \times \left(1 - \frac{3}{7}\right) + \frac{5}{7} \times \left(1 - \frac{4}{7}\right) \times \frac{3}{7} \\
&\quad + \left(1 - \frac{5}{7}\right) \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} \\
&= \frac{209}{343}
\end{aligned}$$

32. (3) Let us assume that A wins after n deuces, $n = 0, 1, 2, 3, \dots$. The probability of a deuce is $(2/3) \times (2/3) + (1/3) \times (1/3) = (5/9)$. [A wins his serve, then B wins his serve or A loses his serve.] So, the probability that A wins game after n deuces is $(5/9)^n \times (2/3) \times (1/3)$. [After n th deuce, A serves and wins, then B serves and loses.] Therefore, the required probability of ' A ' winning the game is

$$\sum_{n=0}^{\infty} \left(\frac{5}{9}\right)^n \times \frac{2}{3} \times \frac{1}{3} = \frac{1}{1 - \frac{5}{9}} \times \frac{2}{9} = \frac{1}{2}$$

33. (2) Die marked with 1, 2, 2, 3, 3, 3 is thrown 3 times.

$$P(1) = \frac{1}{6}, P(2) = \frac{2}{6}, P(3) = \frac{3}{6}$$

$$P(S) = P(4 \text{ or } 6)$$

$$= P(112(3 \text{ cases}) \text{ or } 123(6 \text{ cases}) \text{ or } 222)$$

$$= 3 \times \frac{1}{6} \times \frac{1}{6} \times \frac{2}{6} + 6 \times \frac{1}{6} \times \frac{2}{6} \times \frac{3}{6} + \frac{2}{6} \times \frac{2}{6} \times \frac{2}{6}$$

$$= \frac{6 + 36 + 8}{216} = \frac{50}{216} = \frac{25}{108}$$

34. (2) Let

$$P(S) = P(1 \text{ or } 2) = 1/3$$

$$P(F) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = 2/3$$

$$P(A \text{ wins}) = P[(S S \text{ or } S F S S \text{ or } S F S F S S \text{ or } \dots)]$$

$$\text{or } (F S S \text{ or } F S F S S \text{ or } \dots)]$$

$$\begin{aligned}
&= \frac{1}{1 - \frac{2}{9}} + \frac{2}{1 - \frac{2}{9}} \\
&= \frac{1}{9} \times \frac{9}{7} + \frac{2}{27} \times \frac{9}{7} \\
&= \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}
\end{aligned}$$

$$P(A \text{ winning}) = \frac{5}{21}, P(B \text{ winning}) = \frac{16}{21}$$

35. (1) $P(a) = 0.3$, $P(b) = 0.5$, $P(c) = 0.2$. Hence, a, b, c are exhaustive.

$$P(\text{same horse wins all the three races})$$

$$= P(aaa \text{ or } bbb \text{ or } ccc)$$

$$= (0.3)^3 + (0.5)^3 + (0.2)^3$$

$$= \frac{27 + 125 + 8}{1000} = \frac{160}{1000} = \frac{4}{25}$$

$$P(\text{each horse wins exactly one race})$$

$$= P(abc \text{ or } acb \text{ or } bca \text{ or } bac \text{ or } cab \text{ or } cba)$$

$$= 0.3 \times 0.5 \times 0.2 \times 6 = 0.18 = \frac{9}{50}$$

36. (1) The probability of getting a head in a single toss of a coin is

$p = 1/2$ (say). The probability of getting 5 or 6 in a single throw of a die is $q = 2/6 = 1/3$ (say). Therefore, the required probability is

$$\begin{aligned}
&p + (1-p)(1-q)p + (1-p)(1-q)(1-p)(1-q)p + \dots \\
&= p + (1-p)(1-q)p + (1-p)^2(1-q)^2p + \dots \\
&= \frac{p}{1 - (1-p)(1-q)} \\
&= \frac{1/2}{1 - 1/2 \times 2/3} = \frac{3}{4}
\end{aligned}$$

37. (3) Let the probability that a man aged x dies in a year p . Thus, the probability that a man aged x does not die in a year is $1 - p$. The probability that all n men aged x do not die in a year is $(1 - p)^n$. Therefore, the probability that at least one man dies in a year is $1 - (1 - p)^n$. The probability that out of n men, A_1 dies first is $1/n$. Since this event is independent of the event that at least one man dies in a year, the probability that A_1 dies in the year and he is the first one to die is $1/n [1 - (1 - p)^n]$.

38. (1) For ranked 1 and 2 players to be winners and runners up, respectively, they should not be paired with each other in any round. Therefore, the required probability is $30/31 \times 14/15 \times 6/7 \times 2/3 = 16/31$.

- 39. (1)** Let A denote the event that a sum of 5 occurs, B the event that a sum of 7 occurs, and C the event that neither a sum of 5 nor a sum of 7 occurs. We have,

$$P(A) = \frac{4}{36} = \frac{1}{9}, P(B) = \frac{6}{36} = \frac{1}{6}$$

$$P(C) = \frac{26}{36} = \frac{13}{18}$$

Thus, probability that A occurs before B is

$$\begin{aligned} & P[A \text{ or } (C \cap A) \text{ or } (C \cap C \cap A) \text{ or } \dots] \\ &= P(A) + P(C \cap A) + P(C \cap C \cap A) + \dots \\ &= P(A) + P(C)P(A) + P(C)^2P(A) + \dots \\ &= \frac{1}{9} + \left(\frac{13}{18}\right) \times \frac{1}{9} + \left(\frac{13}{18}\right)^2 \frac{1}{9} + \dots \\ &= \frac{1/9}{1 - 13/18} = \frac{2}{5} \end{aligned}$$

- 40. (4)** Let p_i denote the probability that out of 10 tosses, head occurs i times and no two heads occur consecutively. It is clear that $i > 5$.

For $i = 0$, i.e., no head, $p_0 = 1/2^{10}$.

For $i = 1$, i.e., one head, $p_1 = {}^{10}C_1 (1/2)^1 (1/2)^9 = 10/2^{10}$.

Now for $i = 2$, we have 2 heads and 8 tails. Then, we have 9 possible places for heads. For example, see the construction:

$x T x T x T x T x T x T x T x$

Here x represents possible places for heads. Therefore,

$$p_2 = {}^9C_2 \left(\frac{1}{2}\right)^2 (1/2)^8 = 36/2^{10}$$

Similarly,

$$p_3 = {}^8C_3/2^{10} = 56/2^{10}$$

$$p_4 = {}^7C_4/2^{10} = 35/2^{10}$$

$$p_5 = {}^6C_5/2^{10} = 6/2^{10}$$

$$\begin{aligned} \therefore p &= p_0 + p_1 + p_2 + p_3 + p_4 + p_5 \\ &= \frac{1 + 10 + 36 + 56 + 35 + 6}{2^{10}} = \frac{144}{2^{10}} = \frac{9}{64} \end{aligned}$$

- 41. (4)** According to the given condition,

$${}^nC_3 \left(\frac{1}{2}\right)^n = {}^nC_4 \left(\frac{1}{2}\right)^n,$$

where n is the number of times die is thrown.

$$\therefore {}^nC_3 = {}^nC_4 \Rightarrow n = 7$$

Thus, the required probability is

$$= {}^7C_1 \left(\frac{1}{2}\right)^7 = \frac{7}{2^7} = \frac{7}{128}$$

- 42. (3)** Possibilities of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6).

$$\therefore p = \frac{4}{36} = \frac{1}{9} \text{ and } q = 1 - \frac{1}{9} = \frac{8}{9}$$

Therefore, the required probability is

$${}^3C_2 q^1 p^2 = (3) \left(\frac{8}{9}\right) \left(\frac{1}{9}\right)^2 = \frac{8}{243}$$

- 43. (2)** Here $p = 19/20$, $q = 1/20$, $n = 5$, $r = 5$. The required probability is

$$= {}^5C_5 \left(\frac{19}{20}\right)^5 \left(\frac{1}{20}\right)^0 = \left(\frac{19}{20}\right)^5$$

- 44. (4)** Let X denote the largest number on the 3 tickets drawn.

Then, $P(X \leq 7) = (7/20)^3$ and $P(X \leq 6) = (6/20)^3$. Then, the required probability is

$$P(X = 7) = \left(\frac{7}{20}\right)^3 - \left(\frac{6}{20}\right)^3$$

- 45. (3)** The required probability is

$$\begin{aligned} &= \left[{}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 \right]^2 + \left[{}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 \right]^2 \\ &\quad + \left[{}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \right]^2 + \left[{}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 \right]^2 \\ &\quad + \left[{}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 \right]^2 = \frac{35}{128} \end{aligned}$$

- 46. (3)** The required probability is

1 – Probability of getting equal number of heads and tails

$$= 1 - {}^{2n}C_n \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{2n-n}$$

$$= 1 - \frac{(2n)!}{n!n!} \left(\frac{1}{4}\right)^n$$

$$= 1 - \frac{(2n)!}{(n!)^2} \times \frac{1}{4^n}$$

- 47. (3)** In any trial $P(\text{getting white ball}) = P(W) = 1/2$

$$P(\text{getting black ball}) = P(B) = 1/2$$

Required event will occur if in the first six trial, 3 white balls are drawn in any of the 3 trials from six.

In remaining three trials, black must be drawn.

Three trials in which black balls is drawn can be selected in ${}^6C_3 = 20$ ways.

$$\therefore \text{Required probability} = P(WWWBBB \text{ in any order}) \times P(W)$$

$$= 20 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3 \times \frac{1}{2} = \frac{5}{32}$$

- 48. (4)** Player should get (HT, HT, HT, ...) or (TH, TH, ...) at least $2n$ times. If the sequence starts from first place, then the probability is $1/2^{2n}$ and if starts from any other place, then the probability is $1/2^{2n+1}$. Hence, required probability is

$$2 \left(\frac{1}{2^{2n}} + \frac{m}{2^{2n+1}} \right) = \frac{m+2}{2^{2n}}$$

- 49. (3)** The probability that A gets r heads in three tosses of a coin is

$$P(X = r) = {}^3C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{3-r} = {}^3C_r \left(\frac{1}{2}\right)^3$$

The probability that A and B both get r heads in three tosses of a coin is

$${}^3C_r \left(\frac{1}{2}\right)^3 {}^3C_r \left(\frac{1}{2}\right)^3 = ({}^3C_r)^2 \left(\frac{1}{2}\right)^6$$

Hence, the required probability is

$$\sum_{r=0}^3 ({}^3C_r)^2 \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^6 \{1 + 9 + 9 + 1\} = \frac{20}{64} = \frac{5}{16}$$

- 50. (2)** Let the probability of getting a tail in a single trial be $p = 1/2$. The number of trials be $n = 100$ and the number of trials in 100 trials be X .

We have,

$$\begin{aligned} P(X=r) &= {}^{100}C_r p^r q^{100-r} \\ &= {}^{100}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{100-r} = {}^{100}C_r \left(\frac{1}{2}\right)^{100} \end{aligned}$$

Now,

$$\begin{aligned} P(X=1) + P(X=3) + \dots + P(X=49) \\ &= {}^{100}C_1 \left(\frac{1}{2}\right)^{100} + {}^{100}C_3 \left(\frac{1}{2}\right)^{100} + \dots + {}^{100}C_{49} \left(\frac{1}{2}\right)^{100} \\ &= ({}^{100}C_1 + {}^{100}C_3 + \dots + {}^{100}C_{49}) \left(\frac{1}{2}\right)^{100} \end{aligned}$$

But ${}^{100}C_1 + {}^{100}C_3 + \dots + {}^{100}C_{99} = 2^{99}$

Also, ${}^{100}C_{99} = {}^{100}C_1$

${}^{100}C_{97} = {}^{100}C_3, \dots, {}^{100}C_{51} = {}^{100}C_{49}$

Thus, $2({}^{100}C_1 + {}^{100}C_3 + \dots + {}^{100}C_{49}) = 2^{99}$

or ${}^{100}C_1 + {}^{100}C_3 + \dots + {}^{100}C_{49} = 2^{98}$

Therefore, probability of required event is

$$\frac{2^{98}}{2^{100}} = \frac{1}{4} \quad 2^{98}/2^{100} = 1/4$$

51. (3) In the first 9 throws, we should have three sixes and six non-sixes; and a six in the 10th throw, and thereafter it does not matter whatever face appears. So, the required probability is

$${}^9C_3 \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^6 \times \frac{1}{6} \times 1 \times 1 \times 1 \times \dots \times 1 = \frac{84 \times 5^6}{6^{10}}$$

52. (2) Consider the following events:

A_1 : A speaks truth

A_2 : B speaks truth

Then, $P(A_1) = 60/100 = 3/5$, $P(A_2) = 70/100 = 7/10$.

For the required event, either both of them should speak the truth or both of them should tell a lie. Thus, the required probability is

$$\begin{aligned} &= P((A_1 \cap A_2) \cup (\bar{A}_1 \cap \bar{A}_2)) \\ &= P(A_1 \cap A_2) + P(\bar{A}_1 \cap \bar{A}_2) \\ &= P(A_1)P(A_2) + P(\bar{A}_1)P(\bar{A}_2) \\ &= \frac{3}{5} \times \frac{7}{10} + \left(1 - \frac{3}{5}\right) \left(1 - \frac{7}{10}\right) = 0.54 \end{aligned}$$

53. (4) Probability that one test is held $= 2 \times \frac{1}{5} \times \frac{4}{5} = \frac{8}{25}$

Probability that test is held on both days $= \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

$\therefore$ Probability that the student misses at least one test

$$= \frac{8}{25} + \frac{1}{25} = \frac{9}{25}$$

54. (4) Let A and B, respectively, be the events that urn A and urn B are selected. Let R be the event that the selected ball is red. Since the urn is chosen at random, Therefore,

$$P(A) = P(B) = \frac{1}{2}$$

and $P(R) = P(A)P(R/A) + P(B)P(R/B)$

$$= \frac{1}{2} \times \frac{5}{10} + \frac{1}{2} \times \frac{4}{10} = \frac{9}{20}$$

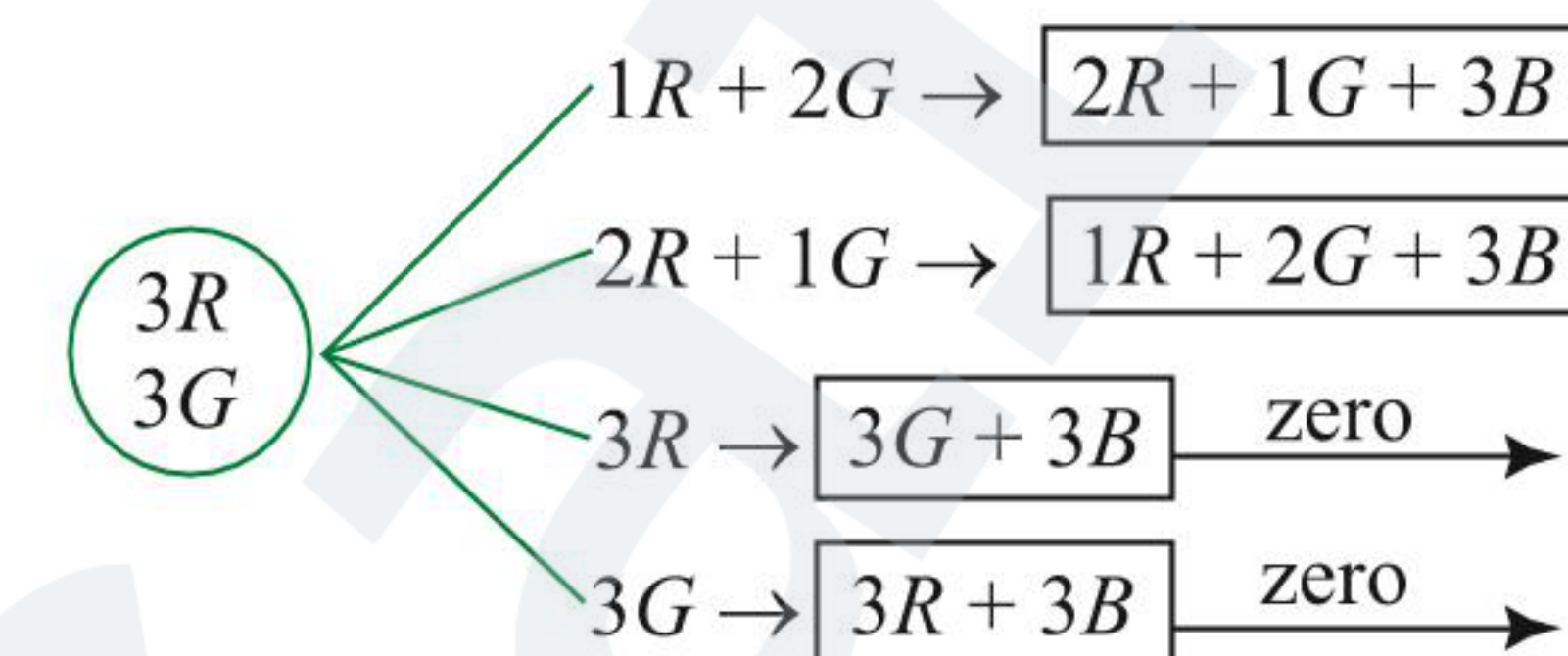
55. (2) $P(4 \text{ biased coin}) = \frac{1}{3}$

$$P(5 \text{ biased coin}) = \frac{1}{4}$$

Hence, the required probability is

$$\begin{aligned} &= \frac{1}{3} \frac{{}^4C_3 {}^{16}C_6}{{}^{20}C_9} + \frac{2}{3} \frac{{}^5C_4 {}^{15}C_5}{{}^{20}C_9} \frac{1}{{}^{11}C_1} \\ &= \frac{2}{33} \left[\frac{{}^{16}C_6 + 5 {}^{15}C_5}{{}^{20}C_9} \right] \end{aligned}$$

56. (3)



The required probability is

$$\begin{aligned} &= \frac{{}^3C_1 {}^3C_2}{{}^6C_3} \frac{{}^2C_1 {}^1C_1 {}^3C_1}{{}^6C_3} + \frac{{}^3C_2 {}^3C_1}{{}^6C_3} \frac{{}^1C_1 {}^2C_1 {}^3C_1}{{}^6C_3} \\ &= 2 \frac{9}{20} \times \frac{6}{20} \\ &= \frac{27}{100} \end{aligned}$$

57. (3) $P(4 \text{ biased coins}) = \frac{1}{3}$

$$P(5 \text{ biased coins}) = \frac{1}{4}$$

The required probability is

$$\begin{aligned} &= \frac{1}{3} \frac{{}^4C_3 {}^{16}C_6}{{}^{20}C_9} \frac{1}{{}^{11}C_1} + \frac{2}{3} \frac{{}^5C_4 {}^{15}C_5}{{}^{20}C_9} \frac{1}{{}^{11}C_1} \\ &= \frac{2}{33} \left[\frac{{}^{16}C_6 + 5 {}^{15}C_5}{{}^{20}C_9} \right] \end{aligned}$$

58. (4) In the first case, the urn contains 3 red and n white balls. The probability that color of both the balls matches is

$$\frac{{}^3C_2 + {}^nC_2}{{}^{n+3}C_2} = \frac{1}{2}$$

$$\text{or } \frac{6 + n(n-1)}{(n+3)(n+2)} = \frac{1}{2}$$

$$\text{or } 2(n^2 - n + 6) = n^2 + 5n + 6$$

$$\text{or } n^2 - 7n + 6 = 0$$

$$\Rightarrow n = 1 \text{ or } 6$$

(1)

In the second case,

$$\frac{3}{n+3} \frac{3}{n+3} + \frac{n}{n+3} \frac{n}{n+3} = \frac{5}{8}$$

Solving, we get

$$n^2 - 10n + 9 = 0$$

$$\Rightarrow n = 9 \text{ or } 1.$$

(2)

From Eqs. (1) and (2), we have $n = 1$.

59. (2) Let $P(m)$, $P(p)$, $P(c)$ be the probability of selecting a book of maths, physics, and chemistry, respectively. Clearly,

$$P(m) = P(p) = P(c) = \frac{1}{3}$$

Again let $P(s_1)$ and $P(s_2)$ be the probability that he solves the first as well as second problem, respectively. Then

$$P(s_1) = P(m) \times P\left(\frac{s_1}{m}\right) + P(p) \times P\left(\frac{s_1}{p}\right) + P(c) \times P\left(\frac{s_1}{c}\right)$$

$$\Rightarrow P(s_1) = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{4}{5} = \frac{19}{30}$$

Similarly,

$$P(s_2) = \frac{1}{3} \times \left(\frac{1}{2}\right)^2 + \frac{1}{3} \times \left(\frac{3}{5}\right)^2 + \frac{1}{3} \times \left(\frac{4}{5}\right)^2 = \frac{125}{300}$$

$$\Rightarrow P\left(\frac{s_2}{s_1}\right) = \frac{\frac{125}{300}}{\frac{19}{30}} = \frac{25}{38}$$

60. (1) $P(A) = P(B) = P(C)$ and $P(A) + P(B) + P(C) = 1$

$$\therefore P(A) = P(B) = P(C) = \frac{1}{3}$$

Also,

$$P(X) = \frac{5}{12}, P(X/A) = \frac{3}{8}, P(X/B) = \frac{1}{4}$$

We have,

$$P(X) = P(A)P(X/A) + P(B)P(X/B) + P(C)P(X/C)$$

$$\therefore \frac{5}{12} = \frac{1}{3} \left\{ \frac{3}{8} + \frac{1}{4} + P(X/C) \right\}$$

$$\Rightarrow P(X/C) = \frac{5}{8}$$

61. (3) Let A denote the event that target is hit when x shells are fired at point I. Let P_1 and P_2 denote the event that the target is at point I and II, respectively. We have $P(P_1) = 8/9$, $P(P_2) = 1/9$, $P(A/P_1) = 1 - (1/2)^x$, $P(A/P_2) = 1 - (1/2)^{55-x}$.

Now from total probability theorem,

$$P(A) = P(P_1)P(A/P_1) + P(P_2)P(A/P_2)$$

$$= \frac{1}{9} \left(8 - 8 \left(\frac{1}{2} \right)^x + 1 - \left(\frac{1}{2} \right)^{55-x} \right)$$

$$= \frac{1}{9} \left(9 - 8 \left(\frac{1}{2} \right)^x - \left(\frac{1}{2} \right)^{55-x} \right)$$

Now,

$$\frac{dP(A)}{dx} = \frac{1}{9} \left(-8 \left(\frac{1}{2} \right)^x \ln \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right)^{55-x} \ln \left(\frac{1}{2} \right) \right)$$

(Note that in this step, it is being assumed that $x \in R^+$)

$$= \frac{1}{9} \ln \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)^{55-x} \left(1 - \left(\frac{1}{2} \right)^{2x-58} \right)$$

$$> 0 \text{ if } x < 29$$

$$< 0 \text{ if } x > 29$$

Therefore, $P(A)$ is maximum at $x = 29$. Thus, 29 shells must be fired at point I.

62. (1) Let E_i denote the event that the bag contains i black and $(10-i)$ white balls ($i = 0, 1, 2, \dots, 10$). Let A denote the event that the three balls drawn at random from the bag are black. We have,

$$P(E_i) = \frac{1}{11} \quad (i = 0, 1, 2, \dots, 10)$$

$$P(A/E_i) = 0 \text{ for } i = 0, 1, 2 \text{ and } P(A/E_i) = \frac{{}^i C_3}{{}^{10-i} C_3} \text{ for } i \geq 3$$

$$\Rightarrow P(A) = \frac{1}{11} \times \frac{1}{{}^{10} C_3} \left[{}^3 C_3 + {}^4 C_3 + \dots + {}^{10} C_3 \right]$$

But

$${}^3 C_3 + {}^4 C_3 + {}^5 C_3 + \dots + {}^{10} C_3 = {}^4 C_4 + {}^4 C_3 + {}^5 C_3 + \dots + {}^{10} C_3$$

$$= {}^5 C_4 + {}^5 C_3 + {}^6 C_3 + \dots + {}^{10} C_3$$

$\vdots$

$$= {}^{11} C_4$$

$$\Rightarrow P(A) = \frac{1}{11} \times \frac{1}{{}^{10} C_3} \times {}^{11} C_4 = \frac{11 \times 10 \times 9 \times 8}{11 \times \frac{10 \times 9 \times 8}{3!}} = \frac{1}{4}$$

$$\therefore P(E_9/A) = \frac{P(E_9)P(A/E_9)}{P(A)} = \frac{\frac{1}{11} \times \frac{{}^9 C_3}{{}^{10} C_3}}{\frac{1}{4}} = \frac{14}{55}$$

63. (2) We define the following events:

A_1 : Selecting a pair of consecutive letters from the word LONDON

A_2 : Selecting a pair of consecutive letters from the word CLIFTON

E : Selecting a pair of letters 'ON'

Then, $P(A_1 \cap E) = 2/5$ as there are 5 pairs of consecutive letters out of which 2 are ON and $P(A_2 \cap E) = 1/6$ as there are 6 pairs of consecutive letters of which 1 is ON. Therefore, the required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}$$

64. (4) A : Doctor finds a rash

B_1 : Child has measles

S : Sick children

$$P(S/F) = 0.9$$

B_2 : Child has flu $\Rightarrow P(B_2) = 9/10$

$$P(S/M) = 0.10$$

$$P(A/B_1) = 0.95$$

$$P(R/M) = 0.95$$

$$P(A/B_2) = 0.08$$

$$P(R/F) = 0.08$$

$$P(B_1/A) = \frac{0.1 \times 0.95}{0.1 \times 0.95 + 0.9 \times 0.08}$$

$$= \frac{0.095}{0.095 + 0.072}$$

$$= \frac{0.095}{0.167} = \frac{95}{167}$$

65. (3) A : car met with an accident

B_1 : driver was alcoholic, $P(B_1) = 1/5$

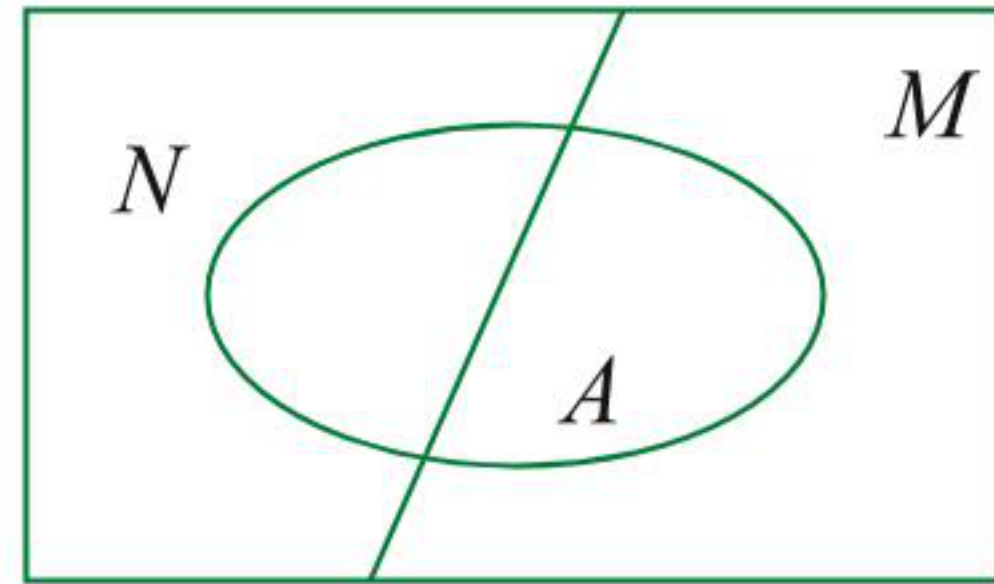
B_2 : driver was sober, $P(B_2) = 4/5$

$$P(A/B_1) = 0.001; P(A/B_2) = 0.0001$$

$$P(B_1/A) = \frac{(0.2)(0.001)}{(0.2)(0.001) + (0.8)(0.0001)} = 5/7$$

66. (1) Let N be the event of picking up a normal die; $P(N) = 1/4$. Let M be the event of picking up a magnetic die; $P(M) = 3/4$. Let A be the event that die shows up 3. Then

$$\begin{aligned} P(A) &= P(A \cap N) + P(A \cap M) \\ &= P(N)P(A/N) + P(M)P(A/M) \\ &= \frac{1}{4} \times \frac{1}{6} + \frac{3}{4} \times \frac{2}{3} = \frac{7}{24} \end{aligned}$$



$$\begin{aligned} P(N/A) &= \frac{P(N \cap A)}{P(A)} \\ &= \frac{(1/4)(1/6)}{7/24} = \frac{1}{7} \end{aligned}$$

67. (1) Consider the following events.

A : ball drawn is black

E_1 : bag I is chosen

E_2 : bag II is chosen

E_3 : bag III is chosen

Then,

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A/E_1) = \frac{3}{5}, P(A/E_2) = \frac{1}{5}, P(A/E_3) = \frac{7}{10}$$

Therefore, the required probability is

$$\begin{aligned} P(E_3/A) &= \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} = \frac{7}{15} \end{aligned}$$

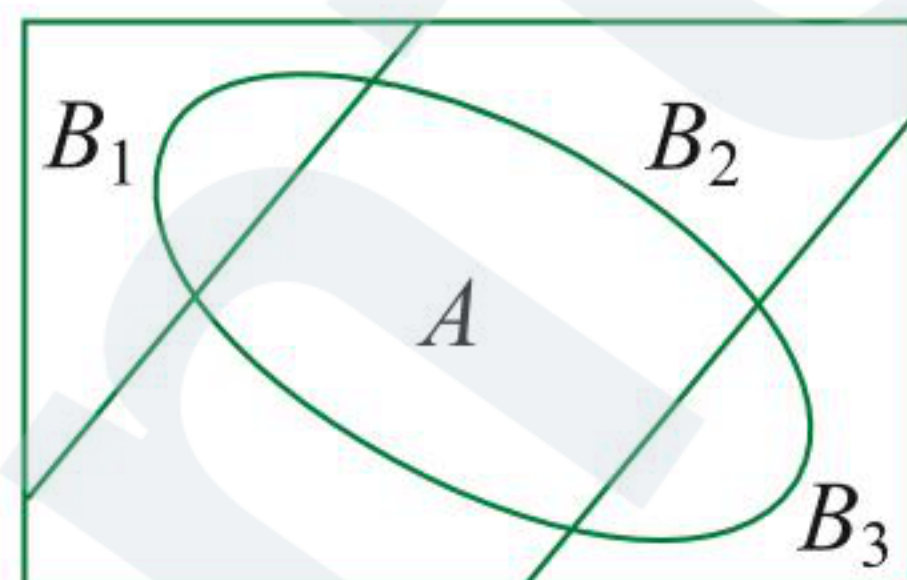
68. (2) Let us consider the following events.

A : card shows up black

B_1 : card with both sides black

B_2 : card with both sides white

B_3 : card with one side white and one black



$$P(B_1) = \frac{2}{10}, P(B_2) = \frac{3}{10}, P(B_3) = \frac{5}{10}$$

$$P(A/B_1) = 1, P(A/B_2) = 0, P(A/B_3) = \frac{1}{2}$$

$$P(B_1/A) = \frac{\frac{2}{10} \times 1}{\frac{2}{10} \times 1 + \frac{3}{10} \times 0 + \frac{5}{10} \times \frac{1}{2}} = \frac{4}{4+5} = \frac{4}{9}$$

Multiple Correct Answers Type

1. (1), (3), (4)

Since A and B are independent events, we have

$$P(A \cap B) = P(A)P(B) = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

$$P(A/B) = P(A) = \frac{1}{2}$$

Now,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{2} + \frac{1}{5} - \frac{1}{10} = \frac{3}{5} \end{aligned}$$

Now,

$$P(A/A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{1/2}{3/5} = \frac{5}{6}$$

$$P[(A \cap B) / (\bar{A} \cup \bar{B})] = P(A \cap B / (\bar{A} \cap \bar{B})) = 0$$

2. (1), (2), (3)

We are given that

$$P(A \cap B') = 0.20, P(A' \cap B) = 0.15, P(A \cap B) = 0.10$$

Now,

$$P(B) = P(A' \cap B) + P(A \cap B) = 0.15 + 0.10 = 0.25$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.10}{0.25} = \frac{2}{5}$$

$$P(A) = P(A \cap B') + P(A \cap B) = 0.3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.55$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1}{3}$$

3. (1), (2), (3)

Let ' H ' be the event that married man watches the show and ' W ' be the probability that married woman watches the show.

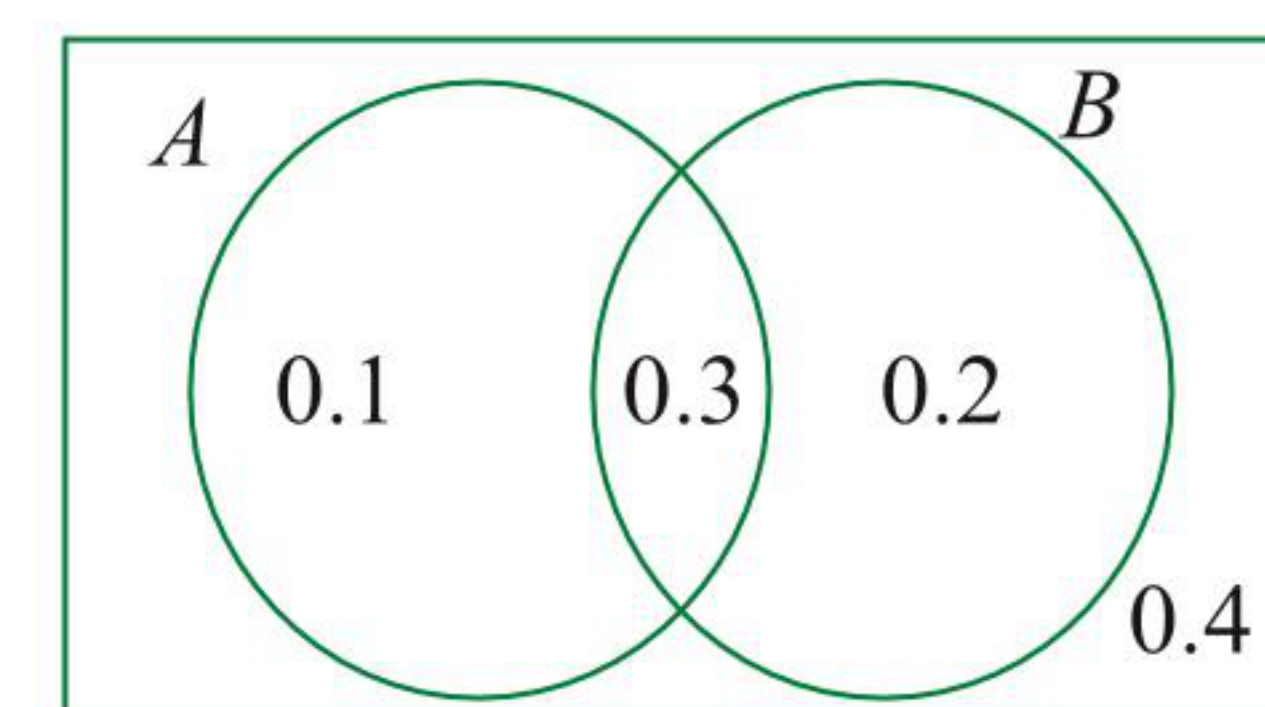
$$\therefore P(H) = 0.4, P(W) = 0.5, P(H/W) = 0.7$$

$$\text{a. } P(H \cap W) = P(W)P(H/W) = 0.5 \times 0.7 = 0.35$$

$$\text{b. } P(W/H) = \frac{P(H \cap W)}{P(H)} = \frac{0.35}{0.4} = \frac{7}{8}$$

$$\begin{aligned} \text{c. } P(H \cup W) &= P(H) + P(W) - P(H \cap W) \\ &= 0.4 + 0.5 - 0.35 = 0.55 \end{aligned}$$

4. (2), (3)



$$P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.3$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6$$

Now,

$$P(E_1) = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.3}{0.6} = \frac{1}{2}$$

$$P(E_2) = \frac{P(A \cap \bar{B}) + P(\bar{A} \cap B)}{P(A \cup B)} = \frac{0.1 + 0.2}{0.6} = \frac{1}{2}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.5} = \frac{3}{5} = 0.60;$$

$$P(B/A) = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$$

$$P(A/(A \cup B)) = \frac{P(A)}{P(A \cup B)} = \frac{0.4}{0.6} = \frac{2}{3}$$

$$\frac{P(B)}{P(A \cup B)} = \frac{0.5}{0.6} = \frac{5}{6}$$

5. (3), (4)

$P(E_1) = 1 - P(\text{unit's place in both is } 1, 2, 3, 4, 6, 7, 8, 9)$

$$P(E_1 = 0 \text{ or } 5) = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$$

$$\begin{aligned}
 P(E_2: 5) &= P(1\ 3\ 5\ 7\ 9) - P(1\ 3\ 7\ 9) \\
 &= \frac{1}{4} - \frac{4}{25} \\
 &= \frac{25-16}{100} = \frac{9}{100}
 \end{aligned}$$

$$\frac{P(E_2)}{P(E_1)} = \frac{9}{100} \times \frac{25}{9} = \frac{1}{4}$$

$$P(E_1) = 4P(E_2)$$

$$P(E_2/E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_2)}{P(E_1)} = \frac{1}{4}$$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_2)}{P(E_2)} = 1$$

6. (1), (2)

The probability that both will be alive for 10 years, hence, i.e., the probability that the man and his wife both will be alive 10 years hence is $0.83 \times 0.87 = 0.7221$. The probability that at least one of them will be alive is

$$\begin{aligned}
 &1 - P\{\text{That none of them remains alive 10 years hence}\} \\
 &= 1 - (1 - 0.83)(1 - 0.87) = 1 - 0.17 \times 0.13 \\
 &= 0.9779
 \end{aligned}$$

7. (2), (3), (4)

(1) False.

$$P(TTT \text{ or } HHH) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$(2) \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B^c)}{1 - P(B)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$P(A \cap B) [1 - P(B)] = P(B) P(A) - P(B) P(A \cap B)$$

$$P(A \cap B) = P(A) P(B)$$

Hence, the given statement is true.

(3) Let E_1 be the white ball drawn in first draw; E_2 be the event that black ball is drawn in second draw; E be the event that white ball is drawn in second draw. Then

$$\begin{aligned}
 P(E) &= P(E/E_1) P(E_1) + P(E/E_2) P(E_2) \\
 &= \frac{d+w}{w+b+d} \left(\frac{w}{w+b} \right) + \frac{w}{w+b+d} \left(\frac{b}{w+b} \right) \\
 &= \left(\frac{w}{w+b} \right) \left(\frac{d+w}{w+b+d} + \frac{b}{w+b+d} \right) \\
 &= \left(\frac{w}{w+b} \right)
 \end{aligned}$$

which is independent of d .

(4) To prove that A, B, C are pairwise independent only. Now,

$$\begin{aligned}
 P(A \cap B) &= P((A \cap B \cap C) \cup (A \cap B \cap \bar{C})) \\
 &= P(A \cap B \cap C) + P(A \cap B \cap \bar{C}) \\
 &= P(A) P(B) P(\bar{C}) + P(A) P(B) P(C) \quad (\text{given}) \\
 &= P(A) \times P(B) [P(C) + P(\bar{C})] \\
 &= P(A) \times P(B)
 \end{aligned}$$

Similarly, for the other two. Hence, this statement is correct.

8. (1), (2), (3), (4)

$$(1) P(E_1) = 1 - P(RRR) = 1 - \left[\frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} \right] = 0.9$$

$$(2) P(E_2) = 3P(BRR) = 3 \times \frac{2}{3} \times \frac{1}{4} \times \frac{2}{5} = 0.2$$

$$(3) P(E_3) = P(RRR/(RRR \cup BBB))$$

$$\begin{aligned}
 &= \frac{P(RRR)}{P(RRR) + P(BBB)} \\
 &= \frac{0.1}{0.1 + \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}} \\
 &= \frac{0.1}{0.1 + 0.4} = 0.2
 \end{aligned}$$

$$(4) P(E_4) = 1 - P(BBB) = 1 - \frac{2}{5} = 0.6$$

9. (1), (2), (3), (4)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{or } \frac{5}{8} = \frac{3}{8} + \frac{4}{8} - P(A \cap B)$$

$$\text{or } P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

Now,

$$\begin{aligned}
 P(A^c/B) &= \frac{P(A^c \cap B)}{P(B)} \\
 &= \frac{P(B) - P(A \cap B)}{P(B)} \\
 &= 1 - 2 \left(\frac{1}{4} \right) \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 2P(A/B^c) &= \frac{2P(A \cap B^c)}{P(B^c)} \\
 &= \frac{2(P(A) - P(A \cap B))}{1 - P(B)} \\
 &= 4 \left(\frac{3}{8} - \frac{2}{8} \right) = \frac{1}{2}
 \end{aligned}$$

Hence, option (1) is correct.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2} = P(B)$$

Hence, option (2) is correct. Again,

$$\begin{aligned}
 P(A^c/B^c) &= \frac{P(A^c \cap B^c)}{P(B^c)} \\
 &= \frac{1 - P(A \cup B)}{1 - P(B)} \\
 &= 2 \left(1 - \frac{5}{8} \right) = \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 P(B/A^c) &= \frac{P(B \cap A^c)}{1 - P(A)} \\
 &= \frac{P(B) - P(A \cap B)}{5/8} \\
 &= \frac{1/2 - 1/4}{5/8} \\
 &= \frac{1}{4} \times \frac{8}{5} \\
 &= \frac{2}{5}
 \end{aligned}$$

Hence,

$$8P(A^c/B^c) = 15P(B/A^c)$$

Hence, option (3) is not correct. Again,

$$2P(A/B^c) = \frac{1}{2}$$

$$\text{or } P(A/B^c) = \frac{1}{4} = P(A \cap B)$$

Hence, option (4) is correct.

10. (1), (2), (4)

We have, the probability that the bomb strikes the target is $p = 1/2$. Let n be the number of bombs which should be dropped to ensure 99% chance or better of completely destroying the target. Then, the probability that out of n bombs at least two bombs strike the target is greater than 0.99. Let X denote the number of bombs striking the target. Then

$$P(X=r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n, r = 0, 1, 2, \dots, n$$

We should have

$$P(X \geq 2) \geq 0.99$$

$$\Rightarrow \{1 - P(X < 2)\} \geq 0.99$$

$$\Rightarrow 1 - \{P(X=0) + P(X=1)\} \geq 0.99$$

$$\Rightarrow 1 - \left\{(1+n)\frac{1}{2^n}\right\} \geq 0.99$$

$$\Rightarrow 0.001 \geq \frac{1+n}{2^n}$$

$$\Rightarrow 2^n > 100 + 100n$$

$$\Rightarrow n \geq 11$$

Thus, the minimum number of bombs is 11.

11. (1), (2), (3)

Option (4) is true if and only if A and B are independent.

12. (1), (2), (3)

$$P(A/B) = P(A) = \frac{1}{2}$$

$$P\{A/(A \cup B)\} = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$$

$$[\because A \cap (A \cup B) = A \cap (A - B - A \cap B)]$$

$$= A - A \cap B - A \cap B = A$$

$$\Rightarrow P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5} - \frac{1}{10}} = \frac{\frac{1}{2}}{\frac{5}{10}} = \frac{5}{6}$$

and similarly,

$$P\left(\frac{A \cap B}{A' \cup B'}\right) = 0$$

13. (2), (3)

Let $P(A) = x$ and $P(B) = y$. Since A and B are independent events, therefore,

$$P(\bar{A} \cap B) = 2/15$$

$$\Rightarrow P(\bar{A})P(B) = 2/15$$

$$\Rightarrow (1 - P(A))P(B) = 2/15$$

$$\Rightarrow (1 - x)y = 2/15$$

(1)

$$\text{and } P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A)P(\bar{B}) = \frac{1}{6}$$

$$\Rightarrow x(1 - y) = \frac{1}{6}$$

$$\Rightarrow x - xy = \frac{1}{6} \quad (2)$$

Subtracting Eq. (1) from Eq. (2), we get

$$x - y = \frac{1}{30} \Rightarrow x = \frac{1}{30} + y$$

Putting this value of x in Eq. (1), we get

$$y - y\left(\frac{1}{30} + y\right) = \frac{2}{15}$$

$$\text{or } 30y - y - 30y^2 = 2/5$$

$$\text{or } 30y^2 - 29y + 4 = 0$$

$$\text{or } (6y - 1)(5y - 4) = 0$$

$$\Rightarrow y = 1/6 \text{ or } y = 4/5$$

$$\Rightarrow P(B) = 1/6 \text{ or } P(B) = 4/5$$

14. (1), (2), (3)

$$(1) P(A) = \frac{1}{5}, P(B) = \frac{7}{25}, P(B/A) = \frac{9}{10}$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{1}{5} + \frac{7}{25} - P(A)P(B/A)\right]$$

$$= 1 - \left[\frac{1}{5} + \frac{7}{25} - \frac{1}{5} \times \frac{7}{25}\right] = \frac{7}{10}$$

$$(2) P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A)P(B/A)}{P(B)}$$

$$= \frac{\frac{1}{5} \times \frac{9}{10}}{\frac{7}{25}}$$

$$= \frac{9}{50} \times \frac{25}{7} = \frac{9}{14} = \frac{18}{28}$$

$$(3) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A)P(B/A)$$

$$= \frac{1}{5} + \frac{7}{25} - \frac{1}{5} \times \frac{9}{10}$$

$$= \frac{10 + 14 - 9}{50}$$

$$= \frac{3}{10}$$

$$(4) P(A' \cup B') = 1 - P(A \cap B)$$

$$= 1 - P(A)P(B/A)$$

$$= 1 - \frac{1}{5} \times \frac{9}{10}$$

$$= \frac{41}{50}$$

15. (1), (4)

The probability that head appears r times is

$${}^{99}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{99-r}$$

which is maximum when $r = 49$ or 50 .

16. (1), (3)

Let one probability of choosing one integer k be $P(k) = \lambda/k^4$. (λ is one constant of proportionality). Then

$$\sum_{k=1}^{2m} \frac{\lambda}{k^4} = 1$$

$$\text{or } \lambda \sum_{k=1}^{2m} \frac{1}{k^4} = 1$$

Let x_1 be the probability of choosing the odd number. Then

$$x_1 = \sum_{k=1}^m P(2k-1) = \lambda \sum_{k=1}^m \frac{1}{(2k-1)^4}$$

Also,

$$\begin{aligned} 1 - x_1 &= \sum_{k=1}^m P(2k) \\ &= \lambda \sum_{k=1}^m \frac{1}{(2k)^4} \\ &< \lambda \sum_{k=1}^m \frac{1}{(2k-1)^4} \end{aligned}$$

$$\Rightarrow 1 - x_1 < x_1$$

$$\Rightarrow x_1 > 1/2$$

$$\Rightarrow x_2 < 1/2$$

17. (1), (2), (3)

Let X = defective and Y = non-defective.

Then all possible outcomes are $\{XX, XY, YX, YY\}$

$$\text{Also, } P(XX) = \frac{50}{100} \times \frac{50}{100} = \frac{1}{4}$$

$$P(XY) = \frac{50}{100} \times \frac{50}{100} = \frac{1}{4}$$

$$P(YX) = \frac{50}{100} \times \frac{50}{100} = \frac{1}{4}$$

$$P(YY) = \frac{50}{100} \times \frac{50}{100} = \frac{1}{4}$$

Here, $A = (XX) \cup (XY)$; $B = (XY) \cup (YY)$; $C = (XX) \cup (YY)$

$$\therefore P(A) = P(XX) + P(XY) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\therefore P(B) = P(XY) + P(YY) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\text{Now, } P(A \times B) = P(XY) = \frac{1}{4} = P(A) \times P(B)$$

Thus, A and B are independent events.

$$P(B \times C) = P(YX) = \frac{1}{4} = P(B) \times P(C)$$

Thus, B and C are independent events.

$$P(C \times A) = P(XY) = \frac{1}{4} = P(C) \times P(A)$$

Thus, C and A are independent events.

$$\begin{aligned} P(A \times B \times C) &= 0 \text{ (impossible events)} \\ &\neq P(A) \times P(B) \times P(C) \end{aligned}$$

Thus, A , B and C are dependent events.

Therefore, we can conclude that A , B , C are pairwise independent but A , B , C are dependent.

Linked Comprehension Type**1. (2), 2. (3), 3. (1)**

Let $P(i)$ be the probability that exactly i students are passing an examination. Now given that

$$P(A_i) = \lambda i^2 \text{ (where } \lambda \text{ is constant)}$$

$$\Rightarrow \sum_{i=1}^{10} P(A_i) = \sum_{i=1}^{10} \lambda i^2 = \lambda \frac{10 \times 11 \times 21}{6} = \lambda \times 385 = 1$$

$$\Rightarrow \lambda = 1/385$$

$$\text{Now, } P(5) = 25/385 = 5/77.$$

Let A represent the event that selected students have passed the examination. Therefore,

$$\begin{aligned} P(A) &= \sum_{i=1}^{10} P(A / A_i) P(A_i) \\ &= \sum_{i=1}^{10} \frac{i}{10} \frac{i^2}{385} \\ &= \frac{1}{3850} \sum_{i=1}^{10} i^3 \\ &= \frac{10^2 \times 11^2}{4 \times 3850} = \frac{11}{14} \end{aligned}$$

Now,

$$\begin{aligned} P(A_1/A) &= \frac{P(A / A_1) P(A_1)}{P(A)} \\ &= \frac{\frac{1}{10} \times \frac{1}{385}}{\frac{11}{14}} \\ &= \frac{1}{11 \times 55} \times \frac{1}{5} \\ &= \frac{1}{3025} \end{aligned}$$

4. (4) Let p_1 be the probability of being an answer correct from section 1. Then $p_1 = 1/5$. Let p_2 be the probability of being an answer correct from section 2. Then $p_2 = 1/15$. Hence, the required probability is $1/5 \times 1/15 = 1/75$.

5. (4) Scoring 10 marks from four questions can be done in $3 + 3 + 3 + 1 = 10$ ways so as to answer 3 questions from section 2 and 1 question from section 1 correctly. Hence, the required probability is

$$\frac{{}^{10}C_3 {}^{10}C_1}{{}^{20}C_4} = \frac{1}{5} \left(\frac{1}{15}\right)^3$$

6. (2) To get 40 marks, he has to answer all questions correctly and its probability is $(1/5)^{10} (1/15)^{10}$. Hence, probability of getting a score less than 40 is

$$1 - \left(\frac{1}{5}\right)^{10} \left(\frac{1}{15}\right)^{10} = 1 - \left(\frac{1}{75}\right)^{10}$$

7. (3), 8. (2), 9. (4)

A : She gets a success

T : She studies 10 h: $P(T) = 0.1$

S : She studies 7 h: $P(S) = 0.2$

F : She studies 4 h: $P(F) = 0.7$

$$P(A/T) = 0.8, P(A/S) = 0.6, P(A/F) = 0.4$$

$$\begin{aligned} P(A) &= P(A \cap T) + P(A \cap S) + P(A \cap F) \\ &= P(T) P(A/T) + P(S) P(A/S) + P(F) P(A/F) \\ &= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4) \\ &= 0.08 + 0.12 + 0.28 = 0.48 \end{aligned}$$

$$= P(F/A) = \frac{P(F \cap A)}{P(A)}$$

$$= \frac{(0.7)(0.4)}{0.48}$$

$$= \frac{0.28}{0.48} = \frac{7}{12}$$

$$P(F/\bar{A}) = \frac{P(F \cap \bar{A})}{P(\bar{A})}$$

$$= \frac{P(F) - P(F \cap A)}{0.52}$$

$$= \frac{(0.7) - 0.28}{0.52}$$

$$= \frac{0.42}{0.52} = \frac{21}{26}$$

10. (2) 11. (1), 12. (3)

$$P(S/T) = \frac{P(S \cap T)}{P(T)}$$

or $0.5 = \frac{P(S \cap T)}{0.69}$

or $P(S \cap T) = 0.5 \times 0.69 = P(S) P(T)$

Therefore, S and T are independent.

$$\begin{aligned} \therefore P(S \text{ and } T) &= P(S)P(T) \\ &= 0.69 \times 0.5 = 0.345 \end{aligned}$$

$$\begin{aligned} P(S \text{ or } T) &= P(S) + P(T) - P(S \cap T) \\ &= 0.5 + 0.69 - 0.345 \\ &= 0.8450 \end{aligned}$$

13. (4) Let E be the event that all the amoeba population dies out;
 E_1 be the event that after first second amoeba splits into two;
 E_2 be the event that after first second amoeba remains the same.
 Then

$$\begin{aligned} P(E) &= P(E_1)P(E/E_1) + P(E_2)P(E/E_2) \\ &= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \\ &= \frac{3}{32} \end{aligned}$$

14. (2) After 2 s, exactly 4 amoeba are alive, i.e., initially amoeba must split into two and in 2nd second again amoeba must split into two. Hence, the required probability is $(1/2) \times (1/2) \times (1/2) = 1/8$.

15. (1) In first second, mother amoeba splits into two with the probability $1/2$.

In 2nd second, two amoeba split into two with probability $(1/2) \times (1/2) = 1/4$

In 3rd second, four amoeba split into two with probability

$$(1/2) \times (1/2) \times (1/2) \times (1/2) = 1/16$$

Hence, the probability that the population is maximum after 3 s is $(1/2) \times (1/4) \times (1/16) = 1/2^7$.

16. (1) $P(A_2) = \frac{18}{36};$

$$P(A_3) = \frac{12}{36} = \frac{1}{3};$$

$$P(A_4) = \frac{9}{36} = \frac{1}{4}$$

$$P(A_5) = \frac{7}{36}$$

$$P(A_6) = \frac{6}{36} = \frac{1}{6}$$

Hence, A_3 is most probable.

17. (3) $P(A_2) = \frac{1}{2}, P(A_3) = \frac{1}{3}, P(A_6) = \frac{1}{6}$

$$\therefore P(A_2 \cap A_3) = P(A_2) P(A_3)$$

$$\Rightarrow P(A_6) = P(A_2) P(A_3)$$

$$\frac{6}{36} = \frac{1}{2} \times \frac{1}{3}$$

Hence, A_2 and A_3 are independent.

18. (4) Note that A_1 is independent with all events $A_1, A_2, A_3, A_4, \dots, A_{12}$. Now, total number of ordered pairs is 23.

$$\underbrace{(1, 1), (1, 2), (1, 3), \dots, (1, 11)}_{22} + (1, 12)$$

Also, A_2, A_3 and A_3, A_2 are independent. Hence, there are 25 ordered pairs.

19. (1) 20. (3), 21. (3)

The scores of n can be reached in the following two mutually exclusive events:

(i) by throwing a head when the score is $(n-1)$;

(ii) by throwing a tail when the score is $(n-2)$

Hence,

$$\begin{aligned} P_n &= P_{n-1} \times \frac{1}{2} + P_{n-2} \times \frac{1}{2} \quad [\because P(\text{head}) = P(\text{tail}) = 1/2] \\ &= \frac{1}{2} [P_{n-1} + P_{n-2}] \end{aligned} \quad (1)$$

$$\Rightarrow P_0 + \frac{1}{2} P_{n-1} = P_{n-1} + \frac{1}{2} P_{n-2}$$

(adding $(1/2) P_{n-1}$ on both sides)

$$= P_{n-2} + \frac{1}{2} P_{n-3}$$

$\vdots$

$$= P_2 + \frac{1}{2} P_1 \quad (2)$$

Now, a score of 1 can be obtained by throwing a head at a single toss. Therefore,

$$P_1 = \frac{1}{2}$$

And a score of 2 can be obtained by throwing either a tail at a single toss or a head at the first toss as well as second toss. Therefore,

$$P_2 = \frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2} \right) = \frac{3}{4}$$

From Eq. (2), we have

$$P_n + \frac{1}{2} P_{n-1} = \frac{3}{4} + \frac{1}{2} \left(\frac{1}{2} \right) = 1$$

$$\text{or } P_n = 1 - \frac{1}{2} P_{n-1}$$

$$\text{or } P_n - \frac{2}{3} = 1 - \frac{1}{2} P_{n-1} - \frac{2}{3}$$

$$\begin{aligned} \text{or } P_n - \frac{2}{3} &= -\frac{1}{2} \left(P_{n-1} - \frac{2}{3} \right) \\ &= \left(-\frac{1}{2} \right)^2 \left(P_{n-2} - \frac{2}{3} \right) \\ &= \left(-\frac{1}{2} \right)^3 \left(P_{n-3} - \frac{2}{3} \right) \\ &= \left(-\frac{1}{2} \right)^{n-1} \left(P_1 - \frac{2}{3} \right) \\ &= \left(-\frac{1}{2} \right)^{n-1} \left(\frac{1}{2} - \frac{2}{3} \right) \\ &= \left(-\frac{1}{2} \right)^{n-1} \left(-\frac{1}{6} \right) \\ &= \left(-\frac{1}{2} \right)^n \frac{1}{3} \end{aligned}$$

$$\text{or } P_n = \frac{2}{3} + \frac{(-1)^n}{2^n} \frac{1}{3} = \frac{1}{3} \left\{ 2 + \frac{(-1)^n}{2^n} \right\}$$

Now,

$$P_{100} = \frac{2}{3} + \frac{1}{3 \times 2^{100}} > \frac{2}{3}$$

$$\text{and } P_{101} = \frac{2}{3} - \frac{1}{3 \times 2^{101}} < \frac{2}{3}$$

$$\Rightarrow P_{101} < \frac{2}{3} < P_{100}$$

Matrix Match Type

1. $a \rightarrow p$; $b \rightarrow r$; $c \rightarrow q$; $d \rightarrow p$.

$$P(A) = \frac{{}^4C_1 {}^8C_2}{{}^{12}C_3} = \frac{4 \cdot 28}{220} = \frac{112}{220} = \frac{28}{55}$$

$$P(B) = \frac{{}^4C_3 + {}^8C_3}{{}^{12}C_3} = \frac{4 + 56}{220} = \frac{60}{220} = \frac{3}{11}$$

$$P(C) = P(WBB \text{ or } BWB \text{ or } WWB \text{ or } BBB)$$

$$\begin{aligned} &= \frac{8}{12} \times \frac{4}{11} \times \frac{3}{10} \times \frac{4}{12} \times \frac{8}{11} \times \frac{3}{10} \times \frac{8}{12} \times \frac{7}{11} \times \frac{4}{10} \times \frac{4}{12} \\ &\quad \times \frac{3}{11} \times \frac{2}{10} \\ &= \frac{96 + 96 + 224 + 24}{12 \times 110} = \frac{440}{12 \times 110} = \frac{1}{3} \end{aligned}$$

$$A \cap B = \phi \Rightarrow A \text{ and } B \text{ are mutually exclusive}$$

$$P(B \cap C) = P(BBB) = \frac{4 \times 3 \times 2}{12 \times 110} = \frac{1}{55}$$

Also,

$$P(B) P(C) = \frac{3}{11} \times \frac{1}{3} = \frac{1}{11}$$

Hence, B and C are neither independent nor mutually exclusive.

$$P(C \cap A) = P(WWB) = \frac{8 \times 7 \times 4}{12 \times 11 \times 10} = \frac{28}{3 \times 55}$$

$$P(C) P(A) = \frac{1}{3} \times \frac{112}{220} = \frac{28}{3 \times 55}$$

Thus, C and A are independent.

Also, A, B, C are mutually exclusive as A and B are mutually exclusive.

2. $a \rightarrow q, r, s$; $b \rightarrow r$; $c \rightarrow p, q$; $d \rightarrow p, q, r, s$.

a. Suppose the coin is tossed n times. The probability of getting head or tail is $1/2$. The probability of not getting any head in n tosses is $(1/2)^n$. The probability of getting at least one head is $1 - (1/2)^n$. Now given that

$$1 - (1/2)^n \geq 0.8$$

$$\text{or } \left(\frac{1}{2} \right)^n \leq 0.2$$

$$\text{or } 2^n \geq 5$$

Therefore, the least value of n is 3.

b. The total number of mappings is n^n . The number of one-one mappings is $n!$. Hence the probability is

$$\frac{n!}{n^n} = \frac{3}{32} = \frac{6}{64} = \frac{3!}{4^3} = \frac{4!}{4^4}$$

Comparing, we get $n = 4$.

c. Given equation is

$$2x^2 + 2mx + m + 1 = 0$$

$$D = 4m^2 - 8(m + 1) \geq 0$$

$$m^2 - 2m - 2 \geq 0$$

$$(m - 1)^2 - 3 \geq 0$$

$$\Rightarrow m = 3, 4, 5, 6, 7, 8, 9, 10$$

Also, the number of ways of choosing m is 10. Therefore, the required probability is $4/5$.

$$\therefore 5k = 4$$

d. $20P^2 - 13P + 2 \leq 0$

$$\Rightarrow (4P - 1)(5P - 2) \leq 0$$

$$\Rightarrow \frac{1}{4} \leq P \leq \frac{2}{5}$$

$$\Rightarrow \frac{1}{4} \leq \frac{1}{5} + \frac{1}{5} \left(\frac{4}{5} \right) + \frac{1}{5} \left(\frac{4}{5} \right)^2 + \cdots + \frac{1}{5} \left(\frac{4}{5} \right)^{n-1} \leq \frac{2}{5}$$

$$\Rightarrow n = 2$$

Hence, maximum as well as minimum value of n is 2.

3. $a \rightarrow r, s$; $b \rightarrow p, q, r, s$; $c \rightarrow p, q, r, s$; $d \rightarrow p$.

a. $P(\text{success}) = 1/2$; $P(\text{failure}) = 1/2$

Suppose ' n ' bombs are to be dropped. Let E be the event that the bridge is destroyed. Then,

$$P(E) = 1 - P(0 \text{ or } 1 \text{ success})$$

$$= 1 - \left(\left(\frac{1}{2} \right)^n + {}^nC_1 \frac{1}{2} \left(\frac{1}{2} \right)^{n-1} \right)$$

$$= 1 - \left(\frac{1}{2^n} + \frac{n}{2^n} \right) \geq 0.9$$

$$\Rightarrow \frac{1}{10} \geq \frac{n+1}{2^n} \text{ or } \frac{2^n}{10(n+1)} \geq 1$$

- b. The bag contains 2 red, 3 white and 5 black balls. Hence $P(S) = 1/5$; $P(F) = 4/5$; Let E be the event of getting a red ball.

$$P(E) = P(S \text{ or } FS \text{ or } FFS \text{ or } \dots) \geq \frac{1}{2}$$

$$\therefore P(F)^n \leq \frac{1}{2}; \left(\frac{4}{5}\right)^n \leq \frac{1}{2}$$

The value of n consistent is 4.

- c. Let there be x red socks and y blue socks and $x > y$. Then

$$\frac{{}^xC_2 + {}^yC_2}{{}^{x+y}C_2} = \frac{1}{2}$$

$$\text{or } \frac{x(x-1) + y(y-1)}{(x+y)(x+y-1)} = \frac{1}{2}$$

Multiplying both sides by $2(x+y)(x+y-1)$ and expanding, we get

$$2x^2 - 2x + 2y^2 - 2y = x^2 + 2xy + y^2 - x - y$$

Rearranging, we have

$$x^2 - 2xy + y^2 = x + y$$

$$\text{or } (x-y)^2 = x + y$$

$$\text{or } |x-y| = x + y$$

Now, $x + y \leq 17$

$$x - y \leq \sqrt{17}$$

As $x - y$ must be an integer, so

$$x - y = 4$$

$$\therefore x + y = 16$$

Adding both together and dividing by 2 yields $x \leq 10$.

- d. Let the number of green socks be $x > 0$. Let E be the event that two socks drawn are of the same colour.

$$P(E) = P(RR \text{ or } BB \text{ or } WW \text{ or } GG)$$

$$= \frac{3}{{}^{6+x}C_2} + \frac{{}^xC_2}{{}^{6+x}C_2}$$

$$= \frac{6}{(x+6)(x+5)} + \frac{x(x-1)}{(x+6)(x+5)}$$

$$= \frac{1}{5}$$

$$\Rightarrow 5(x^2 - x + 6) = x^2 + 11x + 30$$

$$\text{or } 4x^2 - 16x = 0$$

$$\text{or } x = 4$$

4. a $\rightarrow$ q, r; b $\rightarrow$ r; c $\rightarrow$ p; d $\rightarrow$ a, b, c, d.

$$\text{a. } \frac{{}^rC_2}{{}^{r+b}C_2} = \frac{1}{2} \quad 2r(r-1) = (r-b)(r+b-1)$$

$$= 2r(r-1) = (r+b)(r+b-1)$$

$$\text{or } 2r^2 - 2r = r^2 + (2b-1)r + b^2 - 1$$

$$\text{or } r^2 - (1+2b)r + 1 - b^2 = 0$$

$$\text{or } b^2 + 2br + r - r^2 - 1 = 0$$

$$\text{or } b = \frac{-2r \pm \sqrt{4r^2 - 4(r - r^2 - 1)}}{2}$$

$$= -r \pm \sqrt{2r^2 - r + 1}$$

Since b is integer, possible values of r are 3 and 8.

$$\text{b. } {}^4C_2 \left(\frac{r}{r+10} \right)^2 \left(\frac{10}{r+10} \right)^2 = \frac{3}{8}$$

$$\text{c. } \left(\frac{r}{r+10} \right)^2 \left(\frac{10}{r+10} \right)^2 = \frac{1}{16}$$

$$\Rightarrow r = 10$$

- d. Probability of getting exactly n red balls in $2n$ draws is always equal to probability of getting exactly n black balls in $2n$ draws for any value of r and b , hence the ratio r/b can be 10, 3, 8, 2.

5. (2) We have,

$$P(A \cap B) = P(A)P(B) = \frac{1}{12}$$

$$\text{a. } P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2}$$

$$\text{b. } P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)} = \frac{2}{3}$$

$$\text{c. } (C) P\left(\frac{B}{A' \cap B'}\right) = \frac{P(B \cap (A' \cap B'))}{P(A' \cap B')} = \frac{P(\phi)}{P(A' \cap B')} = 0$$

$$\text{d. } P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = P(A') = \frac{2}{3}$$

6. (4) Let E_i denote the event that the bag contains i black and $(12-i)$ white balls ($i = 0, 1, 2, \dots, 12$) and A denote the event that the four balls drawn are all black. Then

$$P(E_i) = \frac{1}{13} \quad (i = 0, 1, 2, \dots, 12)$$

$$P\left(\frac{A}{E_i}\right) = 0 \text{ for } i = 0, 1, 2, 3$$

$$P\left(\frac{A}{E_i}\right) = \frac{{}^iC_4}{{}^{12}C_4} \text{ for } i \geq 4$$

$$\begin{aligned} \text{a. } P(A) &= \sum_{i=0}^{12} P(E_i) P\left(\frac{A}{E_i}\right) \\ &= \frac{1}{13} \times \frac{1}{{}^{12}C_4} [{}^4C_4 + {}^5C_4 + \dots + {}^{12}C_4] \\ &= \frac{{}^{13}C_5}{{}^{13} \times {}^{12}C_4} = \frac{1}{5} \end{aligned}$$

- b. Clearly,

$$P\left(\frac{A}{E_{10}}\right) = \frac{{}^{10}C_4}{{}^{12}C_4} = \frac{14}{33}$$

- c. By Bayes's theorem,

$$\begin{aligned} P\left(\frac{E_{10}}{A}\right) &= \frac{P(E_{10})P\left(\frac{A}{E_{10}}\right)}{P(A)} \\ &= \frac{\frac{1}{13} \times \frac{14}{33}}{\frac{1}{5}} = \frac{70}{429} \end{aligned}$$

- d. Let B denote the probability of drawing 2 white and 2 black balls. Then

$$P\left(\frac{B}{E_i}\right) = 0 \text{ if } i = 0, 1 \text{ or } 11, 12$$

$$P\left(\frac{B}{E_i}\right) = \frac{{}^i C_2 \times {}^{12-i} C_2}{{}^{12} C_4} \text{ for } i = 2, 3, \dots, 10$$

$$\begin{aligned} \therefore P(B) &= \sum_{i=0}^{12} P(E_i) P\left(\frac{B}{E_i}\right) \\ &= \frac{1}{13} \times \frac{1}{{}^{12} C_4} [2\{{}^2 C_2 \times {}^{10} C_2 + {}^3 C_2 + \dots + {}^{10} C_2 \times {}^2 C_2\}] \\ &= \frac{1}{13} \times \frac{1}{{}^{12} C_4} [2\{{}^2 C_2 \times {}^{10} C_2 + {}^3 C_2 \times {}^9 C_2 + \dots + {}^5 C_2 \times {}^7 C_2\} + {}^6 C_2 \times {}^6 C_2] \\ &= \frac{1}{13} \times \frac{1}{495} (1287) \\ &= \frac{1}{5} \end{aligned}$$

Numerical Value Type

1. (7) Let the probability of the faces 1, 3, 5 or 6 be p for each face.

Hence, probability of each of the faces 2 or 4 is $3p$

Now according to the question $4p + 6p = 1$

$$\Rightarrow p = \frac{1}{10}$$

$$\therefore P(1) = P(3) = P(5) = P(6) = \frac{1}{10}$$

$$\text{and } P(2) = P(4) = \frac{3}{10}$$

$\Rightarrow$ Required probability

$$\begin{aligned} p &= P(\text{total of 7 with a draw of dice}) \\ &= P(16, 61, 25, 52, 43, 34) \\ &= 2\left(\frac{1}{10} \cdot \frac{1}{10}\right) + 2\left(\frac{3}{10} \cdot \frac{1}{10}\right) + 2\left(\frac{3}{10} \cdot \frac{1}{10}\right) \\ &= \frac{2+6+6}{100} = \frac{14}{100} = \frac{7}{50} \end{aligned}$$

2. (1) There are n white balls in the turn.

$\Rightarrow$ Probability of Mr. A to draw two balls of same color is

$$\frac{{}^3 C_2 + {}^n C_2}{{}^{n+3} C_2} = \frac{1}{2} \text{ (given)}$$

$$\text{or } \frac{6+n(n-1)}{(n+3)(n+2)} = \frac{1}{2}$$

$$\text{or } n^2 - 7n + 6 = 0$$

$$\Rightarrow n = 1 \text{ or } 6 \quad (1)$$

Also required probability for Mr. B according to the question is

$$\frac{3}{n+3} \cdot \frac{3}{n+3} + \frac{n}{n+3} \cdot \frac{n}{n+3} = \frac{5}{8} \quad (\text{given})$$

$$\text{Solving, we get } n^2 - 10n + 9 = 0; n = 1 \text{ or } 9 \quad (2)$$

From (1) and (2), $n = 1$

3. (2) When A and B are mutually exclusive, $P(A \cap B) = 0$

$$\therefore P(A \cup B) = P(A) + P(B) \quad (1)$$

$$\Rightarrow 0.8 = 0.5 + p$$

$$\text{or } p = 0.3 \quad (2)$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \end{aligned}$$

$$\text{or } 0.8 = 0.5 + p - (0.5)p$$

$$\text{or } 0.3 = p/2$$

$$\text{or } p = 0.6$$

$$\Rightarrow q/p = 2$$

(3)

4. (3) For ranked 1 and 2 players to be winners and runners up, respectively, they should not be paired with each other in any round:

$$\Rightarrow p = \frac{30}{31} \times \frac{14}{15} \times \frac{6}{7} \times \frac{2}{3} = \frac{16}{31}$$

$$5. (6) P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6}{0.8} = \frac{3}{4}$$

(Maximum value of $P(A \cap B) = P(A) = 0.6$)

6. (5) Highest number in three throws is 4, which means at least one of the throws must be equal to 4.

Number of ways when three blocks are filled from $\{1, 2, 3, 4\} = 4^3$

$\therefore$ Number of ways when filled from $\{1, 2, 3\} = 3^3$

$\therefore$ Required number of ways $= 4^3 - 3^3$

$$\therefore \text{Probability, } p = \frac{4^3 - 3^3}{6^3} = \frac{37}{216}$$

7. (4) Let event A : Card is of heart but not king (12 cards)

Event B : King but not heart (3 cards)

Event C : Heart and king (1 card)

$\therefore$ Required probability

$$p = P(E) = \frac{{}^{12} C_1 \cdot {}^3 C_1 + {}^3 C_1 \cdot {}^1 C_1 + {}^1 C_1 \cdot {}^{12} C_1}{{}^{52} C_2} = \frac{2}{52}$$

$$\therefore 104p = 4$$

8. (2) Here $P(E) = \frac{1}{2}$ and $P(F_k) = {}^n C_k \cdot \frac{1}{2^n}$

Also $P(E \cap F_k) = P(\text{exactly } k \text{ heads are obtained and head obtained in first flip})$

$$= \frac{1}{2} \cdot {}^{n-1} C_{k-1} \left(\frac{1}{2}\right)^{n-1}$$

Events E and F_k are independent. Therefore,

$$P(E \cap F_k) = P(E) \cdot P(F_k)$$

$$\text{or } {}^{n-1} C_{k-1} \times \frac{1}{2^n} = \frac{1}{2} \times {}^n C_k \frac{1}{2^n}$$

$$\text{or } 2 \times {}^{n-1} C_{k-1} = {}^n C_k$$

$$\text{or } n = 2k$$

9. (3) $P(E_1) = 1 - [P(\text{all heads}) + P(\text{all tails})]$

$$= 1 - \left[\frac{1}{2^n} + \frac{1}{2^n} \right]$$

$$= 1 - \frac{1}{2^{n-1}}$$

$$P(E_2) = P(\text{no head}) + P(\text{exactly one head})$$

$$= \frac{1}{2^n} + {}^n C_1 \times \frac{1}{2^n} = \frac{n+1}{2^n}$$

$$P(E_1 \cap E_2) = (\text{exactly one head and } (n-1) \text{ tail})$$

$$= {}^n C_1 \times \frac{1}{2} \times \frac{1}{2^{n-1}} = \frac{n}{2^n}$$

If E_1 and E_2 are independent, then

$$\frac{n}{2^n} = \left(1 - \frac{1}{2^{n-1}}\right) \left(\frac{n+1}{2^n}\right)$$

$$\text{or } n = \left(1 - \frac{1}{2^{n-1}}\right)(n+1)$$

$$\text{or } n = n+1 - \frac{n+1}{2^{n-1}}$$

$$\text{or } n+1 = 2^{n-1}$$

$$\text{or } n = 3$$

10. (6) Given S_2 reaches the semifinals.

Since all other players $(2^n - 1)$ are equally likely to win the finals with probability p . We have

$$(2^n - 1)p + \frac{1}{4} = 1$$

$$\text{or } (2^n - 1)p = \frac{3}{4}$$

$$\text{or } p = \frac{3}{4(2^n - 1)}$$

$$\text{If } p = \frac{1}{84}, \text{ then}$$

$$\frac{1}{84} = \frac{3}{4(2^n - 1)}$$

$$\text{or } 2^n - 1 = 63$$

$$\text{or } 2^n = 64$$

$$\text{or } n = 6$$

$$\text{Given } P\left(\frac{D}{T_1}\right) = 10 \cdot P\left(\frac{D}{T_2}\right)$$

$$\text{Let } P\left(\frac{D}{T_2}\right) = x$$

$$\therefore P(T_1) \times P\left(\frac{D}{T_1}\right) + P(T_2) \cdot P\left(\frac{D}{T_2}\right) = P(D) = \frac{7}{100}$$

$$\Rightarrow \frac{1}{5} \times 10x + \frac{4}{5} \times x = \frac{7}{100}$$

$$\Rightarrow x = \frac{1}{40}$$

$$\therefore P\left(\frac{T_2}{D'}\right) = \frac{P\left(\frac{D'}{T_2}\right)P(T_2)}{P\left(\frac{D'}{T_1}\right)P(T_1) + P\left(\frac{D'}{T_2}\right)P(T_2)}$$

$$= \frac{\frac{39}{40} \times \frac{4}{5}}{\frac{30}{40} \times \frac{1}{5} + \frac{39}{40} \times \frac{4}{5}}$$

$$= \frac{\frac{4}{5} \times \frac{39}{40}}{\frac{93}{100}} = \frac{78}{93}$$

Archives

JEE ADVANCED

Singe Correct Answer Type

1. (3) Even G = original signal is green

$E_1 = A$ receives the signal correct

$E_2 = B$ receives the signal correct

E = Signal received by B is green

P (Signal received by B is green)

$$= P(GE_1E_2) + P(G\bar{E}_1\bar{E}_2) + P(\bar{G}E_1\bar{E}_2) + P(\bar{G}\bar{E}_1E_2)$$

$$P(E) = \frac{46}{5 \times 16}$$

$$P(G/E) = \frac{40/5 \times 16}{46/5 \times 16} = \frac{20}{23}$$

2. (1) P (at least one of them solves correctly) = $1 - P$ (none of them solves correctly)

$$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right) = \frac{235}{256}$$

$$3. (3) P(T_1) = \frac{1}{5}$$

$$P(T_2) = \frac{4}{5}$$

$$P(D) = \frac{7}{100},$$

where D is the event that computers produced in the factory turn out to be defective.

4. (2) For coin C_1 ,

Probability of getting head, $P(H) = \frac{2}{3}$

No. of Heads (α)	Probability
0	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
1	$\frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$
2	$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

For coin C_2 ,

Probability of getting head, $P(H) = \frac{1}{3}$

No. of Heads (β)	Probability
0	$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$
1	$\frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$
2	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

For real and equal roots of the equation $x^2 - \alpha x + \beta = 0$,
 $\alpha^2 = 4\beta$

$$\therefore (\alpha, \beta) \equiv (0, 0), (2, 1)$$

$$\text{So, required probability} = \frac{1}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{4}{9} = \frac{20}{81}$$

5. (1)

	S_1	$F_2 = F_1 \cup S_1$	S_2	$G_1 \cup S_2 = G_2$	S_3	Probability
(i)	$\{1, 2\}$	$\{1, 2, 3, 4\}$	$\{1, x\}$	$\{1, 2, 3, 4, 5\}$	$\{1, 2\}$	$P_1 = \frac{{}^2C_2}{{}^3C_2} \cdot \frac{{}^3C_1}{{}^4C_2} \cdot \frac{{}^2C_1}{{}^5C_2} = \frac{1}{60}$
(ii)	$\{2, 3\}$	$\{1, 2, 3, 4\}$	$\{1, x\}$ or $\{x, y\}$	$\{1, 2, 3, 4, 5\}$ or $\{2, 3, 4, 5\}$	$\{2, 3\}$	$P_2 = \frac{1}{3} \left(\frac{{}^3C_1}{{}^4C_2} \times \frac{{}^2C_2}{{}^5C_2} + \frac{{}^3C_2}{{}^4C_2} \times \frac{{}^2C_2}{{}^4C_2} \right) = \frac{2}{45}$
(iii)	$\{1, 3\}$	$\{1, 3, 4\}$	$\{1, x\}$	$\{1, 2, 3, 4, 5\}$	$\{1, 3\}$	$P_3 = \frac{1}{{}^3C_1} \times \frac{{}^2C_1}{{}^3C_2} \times \frac{{}^2C_2}{{}^5C_2} = \frac{1}{45}$

$$\text{Conditional probability} = \frac{P_1}{P_1 + P_2 + P_3} = \frac{1}{5}$$

$$\therefore P(X \cap Y) = \frac{P(Y)}{2} = \frac{2}{5} P(X) = \frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}$$

$$\Rightarrow P(Y) = \frac{4}{15}$$

$$P(X' | Y) = \frac{P(X' \cap Y)}{P(Y)}$$

$$= \frac{P(Y) - P(X \cap Y)}{P(Y)} = \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}} = \frac{2}{4} = \frac{1}{2}$$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$$

Multiple Correct Answers Type

1. (1), (4)

Let $P(E) = e$ and $P(F) = f$

$$P(E \cup F) - P(E \cap F) = \frac{11}{25}$$

$$\Rightarrow e + f - 2ef = \frac{11}{25} \quad (1)$$

$$P(\bar{E} \cap \bar{F}) = \frac{2}{25}$$

$$\Rightarrow (1 - e)(1 - f) = \frac{2}{25}$$

$$\text{or } 1 - e - f + ef = \frac{2}{25} \quad (2)$$

From (1) and (2),

$$ef = \frac{12}{25} \text{ and } e + f = \frac{7}{5}$$

Solving, we get

$$e = \frac{4}{5}, f = \frac{3}{5} \text{ or } e = \frac{3}{5}, f = \frac{4}{5}$$

2. (2), (4)

$$P(X_1) = \frac{1}{2}, P(X_2) = \frac{1}{4}, P(X_3) = \frac{1}{4}$$

$$P(X) = P(X_1 \cap X_2 \cap X_3) + P(X_1 \cap X_2^C \cap X_3) + P(X_1^C \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap X_3^C) = \frac{1}{4}$$

$$(1) P(X_1^C | X) = \frac{P(X \cap X_1^C)}{P(X)} = \frac{1/32}{1/4} = \frac{1}{8}$$

(2) P [exactly two engines of the ship are functioning

$$| X] = \frac{7/32}{1/4} = \frac{7}{8}$$

$$(3) P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)} = \frac{5/32}{1/4} = \frac{5}{8}$$

$$(4) P\left(\frac{X}{X_1}\right) = \frac{P(X \cap X_1)}{P(X_1)} = \frac{7/32}{1/2} = \frac{7}{16}$$

3. (1), (2)

$$\text{We have } P(X) = \frac{1}{3}, \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \text{ and } \frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$$

Linked Comprehension Type

$$1. (1) P(X = 3) = \left(\frac{5}{6}\right) \left(\frac{5}{6}\right) \frac{1}{6} = \frac{25}{216}$$

2. (2) Given that

$$P(X \leq 2) = \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{11}{36}$$

Hence, the required probability is $1 - (11/36) = 25/36$.3. (4) For $X \geq 6$, the probability is

$$\frac{5^5}{6^6} + \frac{5^5}{6^7} + \dots \infty = \frac{5^5}{6^6} \left(\frac{1}{1 - 5/6} \right) = \left(\frac{5}{6} \right)^5$$

For $X > 3$,

$$\frac{5^3}{6^4} + \frac{5^4}{6^5} + \frac{5^5}{6^6} + \dots \infty = \left(\frac{5}{6} \right)^3$$

Hence, the conditional probability is

$$\frac{(5/6)^6}{(5/6)^3} = \frac{25}{36}$$

4. (2) $H \rightarrow$ 1 ball from U_1 to U_2 $T \rightarrow$ 2 ball from U_1 to U_2 E : 1 ball drawn from U_2 $P(W$ from $U_2)$

$$= \frac{1}{2} \times \left(\frac{3}{5} \times 1 \right) + \frac{1}{2} \times \left(\frac{2}{5} \times \frac{1}{2} \right) + \frac{1}{2} \times \left(\frac{{}^3C_2}{{}^5C_2} \times \frac{1}{3} \right) + \frac{1}{2} \times \left(\frac{{}^3C_1 \cdot {}^2C_1}{{}^5C_2} \times \frac{2}{3} \right) = \frac{23}{30}$$

$$5. (4) \quad P\left(\frac{H}{W}\right) = \frac{P(W/H) \times P(H)}{P(W/T) \cdot P(T) + (W/H) \cdot P(H)}$$

$$= \frac{\frac{1}{2} \left(\frac{3}{2} \times 1 + \frac{2}{5} \times \frac{1}{2} \right)}{\frac{23}{30}} = \frac{12}{23}$$

$$6. (1) \quad P(\text{required})$$

$$= P(\text{all are white}) + P(\text{all are red}) + P(\text{all are black})$$

$$= \frac{1}{6} \times \frac{2}{9} \times \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12}$$

$$= \frac{6}{648} + \frac{36}{648} + \frac{40}{648} = \frac{82}{648}$$

7. (4) Let A : one ball is white and other is red

E_1 : both balls are from box B_1 ,

E_2 : both balls are from box B_2 ,

E_3 : both balls are from box B_3

$$\text{Here, } P(\text{required}) = P\left(\frac{E_2}{A}\right)$$

$$= \frac{P\left(\frac{A}{E_2}\right) \cdot P(E_2)}{P\left(\frac{A}{E_1}\right) \cdot P(E_1) + P\left(\frac{A}{E_2}\right) \cdot P(E_2) + P\left(\frac{A}{E_3}\right) \cdot P(E_3)}$$

$$= \frac{\frac{{}^2C_1 \times {}^3C_1}{9C_2} \times \frac{1}{3}}{\frac{{}^1C_1 \times {}^3C_1}{6C_2} \times \frac{1}{3} + \frac{{}^2C_1 \times {}^3C_1}{9C_2} \times \frac{1}{3} + \frac{{}^3C_1 \times {}^4C_1}{12C_2} \times \frac{1}{3}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5} + \frac{1}{6} + \frac{2}{11}} = \frac{55}{181}$$

8. (1), (2)

Box I < Red $\rightarrow n_1$
Black $\rightarrow n_2$

Box II < Red $\rightarrow n_3$
Black $\rightarrow n_4$

$$P(R) = \frac{1}{2} \cdot \frac{n_1}{n_1 + n_2} + \frac{1}{2} \cdot \frac{n_3}{n_3 + n_4}$$

$$P(\text{II/R}) = \frac{\frac{1}{2} \cdot \frac{n_3}{n_3 + n_4}}{\frac{1}{2} \cdot \frac{n_1}{n_1 + n_2} + \frac{1}{2} \cdot \frac{n_3}{n_3 + n_4}}$$

$$= \frac{\frac{n_3}{n_3 + n_4}}{\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4}}$$

by option $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

$$P(\text{II/R}) = \frac{\frac{5}{20}}{\frac{3}{6} + \frac{5}{20}} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{4}} = \frac{1}{4} \times \frac{4}{2+1} = \frac{1}{3}$$

Also, when $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$; $P(\text{II/R}) = \frac{1}{3}$

9. (3), (4)

$$\text{Given } \frac{n_1}{n_1 + n_2} \cdot \frac{n_1 - 1}{n_1 + n_2 - 1} + \frac{n_2}{n_1 + n_2} \cdot \frac{n_1}{n_1 + n_2 - 1} = \frac{1}{3}$$

$$\Rightarrow 3(n_1^2 - n_1 + n_1 n_2) = (n_1 + n_2)(n_1 + n_2 - 1)$$

$$\Rightarrow 3n_1(n_1 + n_2 - 1) = (n_1 + n_2)(n_1 + n_2 - 1)$$

$$\Rightarrow 2n_1 = n_2$$

10. (2) $P(X > Y) = T_1 T_1 + D T_1 + T_1 D$ (where T_1 represents wins and D represents draw)

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{6} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) = \frac{5}{12}$$

$$11. (3) \quad P(X = Y) = DD + T_1 T_2 + T_2 T_1$$

$$= \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right) = \frac{13}{36}$$

Numerical Value Type

1. (6) Let $P(E_1) = x, P(E_2) = y$ and $P(E_3) = z$, then

$$(1-x)(1-y)(1-z) = p$$

$$x(1-y)(1-z) = \alpha,$$

$$(1-x)y(1-z) = \beta,$$

$$(1-x)(1-y)z = \gamma$$

$$\text{So } \frac{1-x}{x} = \frac{p}{\alpha} = x = \frac{\alpha}{\alpha + p},$$

$$\text{Similarly, } z = \frac{\gamma}{\gamma + p}.$$

$$\text{So } \frac{P(E_1)}{P(E_3)} = \frac{\frac{\alpha}{\alpha + p}}{\frac{\gamma}{\gamma + p}} = \frac{\gamma + p}{\alpha} = \frac{1 + \frac{p}{\gamma}}{1 + \frac{p}{\alpha}}$$

$$\text{Also given } \frac{\alpha\beta}{\alpha - 2\beta} = p = \frac{2\beta\gamma}{\beta - 3\gamma} \Rightarrow \beta = \frac{5\alpha\gamma}{\alpha + 4\gamma}$$

Substituting in given relation

$$\left(\alpha - 2\left(\frac{5\alpha\gamma}{\alpha + 4\gamma}\right)\right)p = \frac{\alpha \times 5\alpha\gamma}{\alpha + 4\gamma}$$

$$\text{or } \left(\frac{p}{\gamma} + 1\right) = 6\left(\frac{p}{\alpha} + 1\right)$$

$$\text{or } \frac{\frac{p}{\gamma} + 1}{\frac{p}{\alpha} + 1} = 6.$$

2. (8) Let coin be tossed n times.

$P(\text{at least two heads}) = 1 - P(\text{no heads}) - P(\text{exactly one head})$

$$P(\text{at least two heads}) = 1 - \left(\frac{1}{2}\right)^n - {}^nC_1 \cdot \left(\frac{1}{2}\right)^n \geq 0.96$$

$$\Rightarrow \frac{4}{100} \geq \frac{n+1}{2^n}$$

$$\Rightarrow \frac{2^n}{n+1} \geq 25$$

Therefore, least value of n is 8.

3. (0.50)

$$n(E_2) = \text{arrangement of seven 1's and two 0's} = \frac{9!}{7! 2!} = 36$$

$n(E_1 \cap E_2)$ = both zeros should be in a row or a column

$$\text{like } \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

= (number of ways of arranging 1, 0, 0 in a row) $\times$ (number of ways of arranging such row or column)
 = $3 \times 6 = 18$

$$\therefore P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$$

4. (422.00)

Let $|A| = a$, $|B| = b$, $|A \cap B| = c$
 A and B are independent events.

$$\therefore P(A)P(B) = P(A \cap B)$$

$$\Rightarrow \frac{a}{6} \times \frac{b}{6} = \frac{c}{6} \Rightarrow ab = 6c$$

$(a, b, c) = (3, 2, 1)$	so	${}^6C_1 {}^5C_2 {}^3C_1 = 180$
$= (4, 3, 2)$	so	${}^6C_2 {}^4C_2 {}^2C_1 = 180$
$= (6, 1, 1)$	so	${}^6C_1 = 6$
$= (6, 2, 2)$	so	${}^6C_2 = 15$
$= (6, 3, 3)$	so	${}^6C_3 = 20$
$= (6, 4, 4)$	so	${}^6C_4 = 15$
$= (6, 5, 5)$	so	${}^6C_5 = 6$

5. (6) The probability that in ' n ' trials (hits) there will be exactly ' r '

$$\text{successes, } P(r) = {}^nC_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{n-r}$$

$$\therefore 1 - (P(0) + P(1) + P(2)) \geq 0.95$$

$$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{4}\right)^n - {}^nC_1 \left(\frac{3}{4}\right) \left(\frac{1}{4}\right)^{n-1} - {}^nC_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{n-2} \geq 0.95$$

$$\Rightarrow 1 - \left(\frac{1 + 3n + \frac{9n(n-1)}{2}}{4^n} \right) \geq 0.95$$

$$\Rightarrow 9n^2 - 3n + 2 \leq 0.05 \times 4^n \times 2 \leq \frac{4^n}{10}$$

Thus, the least value of n is 6.

6. (8)

Prime numbers are 2, 3, 5, 7, 11

Sum is prime number = $\{(1, 1), (2, 1), (1, 2), (2, 3), (3, 2), (2, 5), (5, 2), (4, 7), (7, 4), (5, 6), (6, 5)\}$

$$\text{So, } P(\text{Prime}) = \frac{15}{36} = \frac{5}{12}$$

Perfect square numbers are 1, 9.

Sum is perfect square = $\{(1, 3), (3, 1), (2, 2), (4, 5), (5, 4), (6, 3), (3, 6)\}$

$$\text{So, } P(\text{perfect square}) = \frac{7}{36}$$

$$\begin{aligned} \text{Required probability} &= \frac{\frac{4}{36} + \frac{14}{36} \times \frac{4}{36} + \left(\frac{14}{36}\right)^2 \frac{4}{36} + \dots}{\frac{7}{36} + \frac{14}{36} \times \frac{7}{36} + \left(\frac{14}{36}\right)^2 \frac{7}{36} + \dots} \\ &= \frac{\frac{4}{36}}{1 - \frac{14}{36}} = \frac{4}{7} \end{aligned}$$

$$\therefore 14p = 14 \times \frac{4}{7} = 8$$